

Assumed Rates of Discount for Valuations of Publicly Sponsored Defined Benefit Plans

	Abstract	1
I	Introduction	1
	The defined benefit pension contract and the funding rationale	3
	The role of actuarial cost methods and actuarial assumptions	5
	Historical development of the standards for setting assumptions	8
II	Model Approach	11
III	Results	12
	Result 1: Higher interest, lower cost	13
	Result 2: Gen1 advantage re: rate set	13
	Result 3: Gen1 advantage re: benefit increase	15
	Result 4: GenK disadvantage	16
	Result 5: Spreading alters pattern, not direction	18
	Result 6: Advantages independent of actual investments	20
IV	Implications	21
	Magnitude	21
	Governmental accounting and funding for pension costs	21
	Benefit negotiations	23
	Skim funds	24
	Pension obligation bonds	26
V	Counter-Arguments	30
	Expected costs will decline over time	31
	A balance of errors: e.g., salary scale	32
	Mean is acceptable for diversifiable risk, not systemic risk	32
	Equity correlated liabilities, a beta adjustment	34
	Mean reversion	36
	Noise	36
	Population growth	36
VI	Extensions	37
	Corporate Pension Plans	37
	The Virtues of Smoothing	39
VII	Conclusions	40
	Appendices	43
A	ASOP 27, selection of assumptions	43
B	FAS 87, accounting for private pensions	44
C	GASB 25, accounting for public plans	46
D	GASB 27, accounting for public plan sponsors	47
	References	48

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Abstract

Actuaries commonly, and in accordance with professional standards, use expected rates of return (on an anticipated asset mix) to discount the liabilities of defined benefit pension plans and to develop periodic plan expenses and contributions. With risky assets, the symmetry of returns about the expected return is deemed sufficient to develop costs that are unbiased over time. In the public (governmental) plan sector, expenses and contributions are almost always identical and intergenerational equity is a high priority.

This paper uses arbitrage principles to show that the use of expected returns including equity premia is biased in favor of early generations at the expense of later generations, a wealth transfer disguised as risk diversification over time. It is shown that unbiased results can be developed, with no wealth transfers between generations, by assuming nearly risk-free rates of return independently of the actual asset mix.

Because cost computations anticipate equity premia, governments are likely to offer their employees pension benefits and valuable options (Skim funds) at less than their risk-adjusted cost, and to enter into costly strategies such as the issuance of Pension Obligation Bonds (POB's).

I Introduction

Public sector defined benefit (DB) pension plans invest substantially in both equities and fixed income securities. Actuarial Standard of Practice Number 27 (ASOP 27, Appendix A) promulgated by the Actuarial Standards Board says that the actuarial valuation of defined benefit pension plans should take the mix of assets into account when computing plan liabilities and annual costs and contributions. Assumed investment returns and discount rates must fall into a best-estimate range defined around the expected return on the anticipated asset allocation. Thus the resulting rate is usually well above the risk-free rate for the life of the pension liabilities.

The actuarial literature, including ASOP 27, justifies the use of expected return as an unbiased estimate of the rates of return that will be earned by the plan assets over the life of the liabilities. It is understood that actual returns will deviate from those expected

and that better-than-expected returns in some years will offset worse-than-expected returns in others. It is also anticipated that, in future years, expected rates of return will vary and that plan actuaries will then change the assumed rate of return accordingly.

An immediate implication of the procedure described above is that plans that invest more heavily in equities require lower levels of funding and that these lower levels usually may be realized immediately upon setting the asset allocation. Although it is recognized that higher expected rates of return must be associated with greater uncertainty with respect to future returns, year-by-year valuations incorporate no explicit recognition of these differing levels of risk.

This paper uses the lessons of modern finance to criticize the actuarial process for determining annual costs for public sector pension plans. I argue that the use of expected return without adjustment for risk constitutes a wealth transfer among generations with the current generation enjoying the benefit of risks that must be borne by later generations. It is shown that, although the actuarial assumption employed is unbiased with respect to anticipated returns, it is not unbiased with respect to risks. The intergenerational transfers may be represented as a borrowing by the first generation (Gen1) upon which interest is paid by all future generations (GenK, $K > 1$) with the principal repayment left as an additional onus for the final generation (Gen Ω).

Actuarial principles, as contrasted with actuarial assumptions under ASOP 27, support risk recognition. This paper argues that the current prescription is flawed. Regardless of the actual allocation of assets, the risk-adjusted unbiased plan cost is computed using risk-free rates applicable to the time horizons of the pension liabilities.

This paper addresses public sector pension plans. Very similar arguments may be applied to defined benefit pension plans maintained by private corporations. ASOP 27 applies to both public and private defined benefit pension plans. The decision to focus on the public sector is derived from the “cleaner laboratory” presented in the public area. The cleanliness of the laboratory arises from a common practice in the public sector, wherein annual contributions to the plan and accounting expenses for the sponsor are almost always identical. The word *costs* is used throughout the paper to refer to either or both of these concepts except when greater specificity is necessary. Neither the

accounting rules of the Financial Accounting Standards Board (FASB) nor the statutes and regulations of the Employee Retirement Income Security Act (ERISA) are applied in the public area (although less restrictive Governmental Accounting Standards Board (GASB) accounting rules often apply as well as state and local statutes). As a consequence, most actuaries practicing in the public sector are able to work with a single set of books of account. Another aspect of the cleaner laboratory is that the tax-exempt status of pension trusts matches the nontaxability of governmental entities in contrast to the taxability of corporations

The remainder of this section discusses the defined benefit contract, the funding rationale, the role of *Actuarial Cost Methods* (ACM's) and *Actuarial Assumptions*, and the historical development of the standards for setting assumptions.

The defined benefit pension contract and the funding rationale

Defined benefit pension plans implement long-term contracts between the plan sponsor and its employees, under which employees accept some of their total compensation in the form of promised annuities commencing after employment ends. The contract terms are frequently complicated and may embed options, forfeitures, qualifying age and service rules, etc. The group nature of the contract makes it necessarily true that an individual employee almost never receives a promise set¹ that matches the economic value of his or her particular wage concession. This may be thought of as a socialization principle or, if individual economic equity is a high priority for the reader, it may be assumed that the total ex-ante compensation varies with the individual circumstances of the employee.

It is a common conceptual error to assume that the conceded wages form a fund that is invested on behalf of the employees. Part of this misconception arises from the legal definitions with respect to the responsibilities of the plan's trustees². More arises from

¹ For every plan participant and at every point in time, there exists a set of contingent benefits that may vary by age, future employment, future pay, marital status, etc. The entire set is promised to the employee by the defined benefit pension contract.

² There is a colloquial understanding that a fiduciary must cause the plan to be administered in the best interests of the plan participants. More accurately, the plan must be administered so that the employee promises are protected. There is no mandate to act in any fashion to expand the promises beyond those made by the sponsor.

the negotiating process wherein total group concessions may equate to some (perhaps miscalculated) measure of employer costs. Further, some very poorly informed employees may confuse defined benefit and defined contribution arrangements.

It is conceptually more accurate to view the DB contract as an exchange of part of total current compensation for a promise set. The fact that many plans based in the United States do accumulate assets does not mean that these assets should be viewed as an accumulating investment of deferred wages.

In the case of public DB plans which, unlike most private plans, frequently require employee contributions, the assets-as-deferred-wages concept is harder to shed. A corporate finance analogy may help. The promise made to bondholders of a corporation is fixed regardless of the performance of corporate assets. When default is an issue, the promise may be subject to contingencies, but the promise itself does not include a share of asset performance beyond that necessary to deliver the contractual amount. This is true despite the fact that bondholders may have supplied the very funds that purchased the assets. Similarly, contributing employees of a public fund are entitled to fixed promises in exchange for reduced salaries as well as for that portion of their salaries that comprises the contribution requirement³.

The plan assets should be viewed as a segregated portion of the assets of the sponsor, set aside to serve particular purposes. This segregation is strong; these assets are unreachable for other purposes until all the plan obligations have been satisfied. In the private sector, ERISA makes funding a requirement in order to provide collateral for long-term promises made by a knowledgeable (but not necessarily financially strong) employer to vulnerable employees. Further, under ERISA, where the Pension Benefit Guaranty Corporation is inserted into the gap between promises made and promises kept, funding reduces the risks and some of the moral hazard associated with governmental guarantees of private promises.

³ One might ask why have employee contributions rather than just lower pay? The inclusion of employee contributions has achieved various purposes historically. It allows higher nominal pay rates to be offered to prospective employees, persuades taxpayers that they do not bear the full cost, provides backloading of benefits in favor of longer service older employees, commits employees to the success of the plan and strengthens the bargaining position of employee representatives with respect to investments and benefits, etc.

In the public sector, it has been argued that, because of the taxing power of most sponsors and because accounting expenses are almost always identical to the amount of funds contributed, the primary rationale for funding is to provide an opportunity for a system of intergenerational equity (Peskin, 1999). This paper will show that the actuarial processes employed squander this opportunity.

The role of actuarial cost methods and actuarial assumptions

Because the operation of a defined benefit plan implements a long-term exchange contract, and because terms of the exchange are subject to periodic updating, it is necessary to employ actuarial valuations as a navigational tool through time.

Actuarial valuations are the periodic (almost always annual) financial analyses of the assets and liabilities of a pension plan leading to the development of a schedule (a vector) of future plan costs. Every valuation employs an Actuarial Cost Method (ACM), a set of data, and a set of actuarial assumptions.

The cost method is the procedure that develops the future periodic costs vector, subject to the constraint that the accumulation of actuarial assets, future contributions and investment returns is exactly sufficient to meet all benefits when due:

$$\text{Present Value of Future Benefits (PVFB)} = \text{Actuarial Asset Value (AAV)} + \text{Present Value of Future Costs (PVFC)}$$

Benefits are projected using assumptions about deaths, retirements, terminations of employment, salaries and such other factors as the plan may use to define the benefits that it will pay. The vector of projected benefits is then discounted at the assumed interest rate to yield the PVFB. Note that the “closed group” valuations used to develop plan costs do not include future hires. Special “forecast” valuations may incorporate future hires⁴. For a discussion of the particularities of various cost methods, see Berin (1976), McGill et al (1996), Trowbridge (1952), Trowbridge and Farr (1976).

⁴ McGill et al (1996), pp. 272

The computation of the Actuarial Asset Value is defined as part of the ACM. ERISA, FASB and GASB all require that the AAV be “market-related”. Typically, the AAV is a weighted average of recent market values subject to a market-based minimum and maximum. The use of the AAV in lieu of the market value of the assets has two implications. The first is that asset realization is effectively smoothed to a greater extent than is the realization of other actuarial assumptions. For the purposes of this paper, this is a matter of degree and not of direction. All smoothing activities are modeled herein as a spreading of the difference between realization and expectation over some future period of time. The second implication is addressed in GASB 25 paragraph 36c which indicates that the assumed interest rate should be developed with consideration given to the basis used to determine the AAV. This may be understood to suggest that when the AAV is greater (lesser) than the market value of the assets, the actuary will reduce (increase) any interest rate assumed for the future from that which might be deduced from the current state of the market alone. As noted in Section IV, this suggested understanding is weakly communicated and may often be ignored in practice

The PVFC is the balancing item in the constraint above. Since the AAV is not itself a function of the assumed discount rate, the direction of change in the PVFC will follow the direction of change in the PVFB. As a result, the PVFC will rise or fall as the assumed discount rate is decreased or increased.

An integral part of every ACM is the treatment of gains or losses (Dreher, 1960) so that the constraint equation is self-correcting and valid in every period. Also inherent in every method is a procedure for translating the PVFC into a vector of projected (scheduled) future costs. The differences in ACM’s are primarily differences in how these two functions are performed.

Aggregate methods generally divide the PVFC into two pieces: the Present Value of Future Normal Costs (PVFNC) and the Unfunded Accrued Liability (UAL)⁵. The prescription for this division is an element of the ACM and for the purposes of this paper may be treated as arbitrary. The PVFNC is divided by the PV Future Payroll to get a percentage which is then applied to the future pay vector to determine each future year’s

expected normal cost. The UAL is often amortized by dividing it by an annuity certain to generate a level dollar amount per year for the period certain. GASB additionally recognizes a method that amortizes the UAL over future payrolls projected for a certain period⁶. The cost vector may then be computed as the amortization vector plus the product of the normal cost percentage and the projected future pay vector.

Accrued methods also develop a UAL and amortize it. The accrued methods develop the normal cost as the PV of benefits deemed to be accrued in each future year. This contrasts with the aggregate methods in that future normal costs are not expected to be a constant percentage of the projected payroll.

The process for allocating the PVFC over future periods (in essence, the ACM) incorporates historic notions about the “right” way to recognize the impact of accruing pension liabilities over the working lifetimes of employees. The accrued benefit methods may be understood to follow a “yearly renewable term” premium structure borrowed from life insurance contracts. The aggregate methods follow a structure based on “level premium life”. Some methods level the annual costs individual by individual (individual methods) and then sum them to get values for the group, while others (aggregate methods) develop level (as a percentage of payroll) costs for the group directly.

For our purposes, it is sufficient to observe that every method allocates the PVFC over time somewhat arbitrarily. Because some of the allocation procedures use interest sensitive measures (e.g., PVFP) in the denominator of some of the calculations, the inverse relationship between the discount rate and the PVFC is not always transmitted to the first term (the current year’s cost) of the cost vector. It may be shown that no ACM is immune to this “pathology” given an unusual combination of data and assumptions. Nonetheless, it is almost always the case that the current year’s cost is inversely related to the assumed discount rate.

⁵ Under the traditional “Aggregate” method, the UAL is defined to be zero.

⁶ GASB 25, paragraph 36f3

Historical development of the standards for setting assumptions

Appendix A summarizes those provisions of Actuarial Standard of Practice 27 that pertain to setting assumed rates of discount. ASOP 27 is firmly rooted in actuarial processes that were developed during the 1960's as pension plans began to include substantial investments in equities.

Prior to that time, many pension plans were insured and most pension plans invested almost exclusively in fixed income securities. With stable interest rates prevailing in the post-War years, and with commercial computers of a rudimentary sort not arriving until the late 1950's, most actuaries carried fixed income investments at book value and adjusted discount rates very infrequently. The ACM's, which were developed somewhat earlier and summarized by Trowbridge (1952), were designed to provide a smooth and self-correcting accumulation of assets sufficient to meet plan liabilities.

During the 1950's and 1960's, a movement to unbundle pension management gained momentum. Plans were taken out of insurance companies. Uninsured, "trusteed", plans required investment, actuarial and administrative services that were frequently delivered by independent parties. A major virtue of the trusteed plans was their ability to invest in equities. A seeming drawback was the volatility that this might impose upon annual pension costs. Even though the ACM's described by Trowbridge already smoothed actuarial gains and losses over many years, the traditional book value treatment was not an adequate approach to valuing equities and particularly for actively managed equities.⁷ Various ways of defining the actuarial value of assets were developed (Hamilton and Jackson, 1968). Schemes that balanced the recognition of equity returns against the desired smoothness of costs gained favor.

The priority of smoothness and the expectation that equity would reduce costs in the long run led naturally to the use of assumed discount rates that included premia for the expected returns on equities in excess of that expected for fixed income investments.

⁷ Note that the book value of an equity portfolio is essentially a trailing average of the market value of each security on its acquisition date; active turnover would cause book value to approach market value; inactive management would leave equities at outdated values.

Contemporaneously with the development of actuarial processes to derive pension costs (essentially an accounting methodology), economists looking at the securities markets began to develop financial economics. Modigliani and Miller (1958) applied the arbitrage concept to assert that an investment that may be replicated by another must, absent arbitrage, carry the same price. Although the use of expected returns unadjusted for risk violates this proposition, there seemed to be little reason for actuaries to adjust a thirty-year funding plan to incorporate lessons that, even if then known to the actuaries, would appear to apply to the short-lived prices at which securities were traded.

Various accounting opinions and IRS rulings codified the actuarial processes in the years prior to the enactment of the Employee Retirement Income Security Act (1974). The primary accounting opinion was Opinion No. 8 of the Accounting Principles Board (APB 8, 1966) which identified mainstream ACM's and provided guidance with respect to the periods over which unfunded liabilities and actuarial gains and losses had to be amortized. IRS rulings also identified the major generic ACM's and provided limitations on the number of years over which unfunded liabilities could be funded on a tax-deductible basis. Although the IRS rules applied to amounts contributed to plans and APB 8 applied to the sponsors' books of account, most employers could contribute and account using the same numbers.

ERISA provided further formal (statutory) codification of the rules that the IRS had developed on a slimmer statutory footing in earlier years. ERISA also added the Pension Benefit Guaranty Corporation as an insurer of employee pensions promised by companies that were unable to meet the promises, minimum funding standards based on common ACM's and limited amortization periods, codification of pension-specific fiduciary standards, and the rule that actuarial assumptions had to be "reasonable" in the aggregate. This latter rule meant that a conscious underestimate of the discount rate might be offset by a consistent underestimate of future pay increases or that low rates of assumed mortality might be offset by low estimates of plan expenses. There was no consideration given to the appropriateness, or lack thereof, of the expected-returns approach to setting the discount rate.

The use of offsetting assumptions predated and survived ERISA. From an economist's perspective, the use of offsetting assumptions makes comparative statics unreliable.

Even though the plans might be in some long-term balance and even though the annual cost might be, in some sense, “right”, the marginal cost relationships are distorted by this offset process. This distortion has at least two major implications:

- The marginal cost of benefit changes, including optional forms of benefits, is only likely to be comparable to a change in current wages (for which such benefits are often exchanged in negotiations) by accident.
- Investment products may be designed to allow plan managers to achieve their desired goals regardless of the merits of the investments. Opaque methodologies and inaccurate marginal costs encourage manipulation by plan managers to the potential detriment of employees or principals (taxpayers or shareholders).

In 1985 the Financial Accounting Standards Board (successor to the APB) approved and promulgated FAS 87 which became effective for fiscal years beginning after December 15, 1986. This standard superseded FAS 36 which had become effective in 1980 as a post-ERISA successor to APB 8. Unlike ERISA whose cost rules apply entirely to plan contributions, FAS 87 applies entirely to employer accounting for pension costs.

The provisions of FAS 87 that are most pertinent to this paper are briefly summarized in Appendix B. FAS 87 introduced the idea that each significant assumption must “reflect the best estimate of the plan’s future experience solely with respect to that plan assumption”⁸. In those instances where the subject is corporate books of account, the use of offsetting assumptions no longer applied. FAS 87 moved the compromise between financial reality and accounting smoothness towards the former despite the objections of the advocates of the latter.

How the provisions of FAS 87 reinforce or mitigate the arguments of this paper as they might be applied to the private plan sector is discussed in Section VI.

The plans that constitute the primary subject of this paper are governed neither by ERISA nor by FAS 87. Public plans are governed by the statutes of states and municipalities and, for some federally sponsored plans, by federal law. The accounting

⁸ FAS 87, paragraph 191.

for such plans is the province of the Governmental Accounting Standards Board (GASB) a sister organization of FASB under the umbrella of the Financial Accounting Foundation. After circulating some FAS 87-like proposals within the public plan community, the GASB released in November, 1994, GASB 25 which applies to the books of account of the public pension plan and GASB 27 which applies to the sponsor books of account. Despite the later publication date, these rulings are more like the FAS 35/36 updates of APB 8 than they are like FAS 87. In particular, they recognize all the common ACM's and allow substantial flexibility with respect to amortization periods. They defer to ASOP 27 (as a successor to ASOP 4) with respect to the setting of discount rates, which are not distinguished from expected returns on plan assets. Thus the GASB statements incorporate the use of expected returns that include estimated risk premia for equity and other investments. The statements require that each assumption's reasonableness "should be considered independently based on its own merits, its consistency with each other assumption, and the combined impact of all assumptions."⁹ This appears to be similar to the FAS 87 admonition that each assumption be developed without contemplation of offsets.

ASOP 27 (1996) is the latest pronouncement by the actuarial profession with respect to the setting of the economic assumptions for use in valuing defined benefit pension plans. It replaces that part of ASOP 4 (last revised in 1993) that deals with economic assumptions. ASOP 27 refines, but does not correct, earlier actuarial attempts to apply the lessons of finance to the selection of economic assumptions.

II Model Approach

Actuarial Cost Methods are accounting procedures used to develop periodic (annual) pension costs. The costs so developed may be used to account for the expenses of the plan sponsor or as a budgeting plan for contributions to the plan by its sponsor, or both.

The methods incorporate complex details which vary from method to method. Strictly modeling each method in order to demonstrate the thesis of this paper would surely be to lose the forest for the trees. This paper will, instead, rely on a simpler generic model containing the critical common elements of all ACM's. This model is built in steps in

⁹ GASB 25, paragraph 36b.

Section III. It includes: the promise of a defined benefit made during the working lifetimes of plan participants; the discounting of the promise for the time value of money; the concept of gain and loss arising from the distinction between expected returns on risky assets and the realized returns thereon; and the spreading of such gains and losses over periods that exceed the time over which any particular gain or loss arises. The model does not deal with demographic assumptions and variations in their realization, taking these assumptions as perfect forecasts. Furthermore, the model includes an implicit assumption that defined benefits are accurately modeled as nominal bond-like promises. Section V deals with the more general case where promises include components correlated with the excess returns on equities.

The model marks time in “periods” of unspecified length. For the purposes of the Results other than Result 5, it is appropriate to recognize that a period might be roughly in the range of twenty to forty years. Thus risk-free returns would be represented by long bonds. Equity returns would be represented by total returns on an index using models that assume neither mean-reversion nor trend-following, implying that equity risk is persistent and grows with the square-root of the length of the period. With the duration of plan liabilities frequently as great as fifteen years, the use of an expected return on equities for a fund half invested in equities will result in a liability roughly one-third less than a liability computed using risk-free rates.

For Result 5, where spreading risks over multiple time periods is the focus, it is appropriate to think of annual periods with gains and losses being actuarially spread over fifteen to thirty one-year periods.

III Results

This section develops a simple generic model of the actuarial treatment of investment risk and the spreading thereof across generations of taxpayers. The first result provides the sign of the derivative of actuarial cost with respect to the interest rate assumption and observes that some rare combinations of methods, assumptions and data may contradict the general rule. The later results develop the model.

Result 1: The use of higher interest rate assumptions always leads to a reduction in the present value of future costs and *almost always* lowers this year's actuarial cost.

It would seem to be self-evident that raising the assumed interest rate would always lower this year's cost. It is a peculiar aspect of actuarial cost methods that this statement is not true for any of the common actuarial cost methods¹⁰.

As outlined in Section I, the Present Value of Future Costs is inversely related to the assumed discount rate. This is not surprising. What is surprising is that the first entry in the cost vector is not necessarily decreasing in the interest rate. While the cases where this first (or current) period cost does not decrease with rising interest rate assumptions are rare and may be considered to be, in some sense, pathological, it is not clear whether the pathology is in the unusual data sets that cause this effect (frequently comprised of workforces where most of the actively employed are close to retirement age) or in the weak design and arbitrary nature of the actuarial methods.

Having noted that this year's costs generally, but not always, decline with rising assumed interest rates, we will, hereinafter, address only those well-behaved cases that represent all but a miniscule fraction of defined benefit pension plans.

Result 2: Adjusting the assumed interest rate to reflect a change in equity allocation upward is a windfall for the generation that does it.

ASOP 27 indicates that the marginal effect of an increase in the anticipated equity allocation should be an increase in the assumed interest rate. Assuming that Result 1 is effective, any increase in the assumed interest rate will lower this year's cost independently of whether it is accompanied (justified?) by a change in the asset allocation. In the sense that risk premia are earned entirely by the bearers of the risk, such a decrease in current costs cannot be earned by the generation that changes the interest rate assumption even if the asset allocation is changed as well. All of the

¹⁰ The statement is true for the special case of the FAS 87 pension expense when only the expected return on assets is increased.

increased risk associated with the greater equity exposure will be faced by future generations yet some of the risk premium is being expropriated by this generation.

The following example will serve to illustrate this point and to initiate the model that will be used to evaluate later results as well. Define:

α = fraction invested in equities; $1 - \alpha$ = fraction invested in riskless asset

r = riskless rate of return ¹¹

\bar{q} = expected return on equities; \tilde{q} = actual return on equities

\bar{e} = expected return on asset mix = $\alpha \bar{q} + (1 - \alpha)r$; \tilde{e} = actual return on asset mix = $\alpha \tilde{q} + (1 - \alpha)r$

Employees of the municipal system work for one period and are retired in the following period at which time they receive a fixed sum promised to them at the beginning of the employment period. Taxpayers live for two periods. In the first period, they are endowed with assets that will be used for their expenses in the second period and which they invest in the securities markets during period one. In the second period, they consume all of their assets net of taxes. The tax obligation includes an amount that will be invested by the municipal pension plan in order to meet the retirement benefits of those municipal employees whose work period is contemporaneous with the consumption period of the taxpayers. Taxpayers are also obligated to make up any actuarial losses (shortfall) or receive any actuarial gains (longfall) left by the previous generation of taxpayers. The populations of taxpayers and municipal employees are stationary and the benefits promised to retirees translate into \$1 per taxpayer. Thus:

Generation 1 of taxpayers (Gen1) promises to pay \$1 in period two to municipal employees of period one. Gen1 sets aside (“funds”, used as a verb) this promise with \$1 discounted for one period. Gen2 promises and funds for \$1 payable in period three and is held responsible for paying \$1 in period two using the investment proceeds of the Gen1 funding.

¹¹ The term structure of risk-free rates applicable to the liability cash flows.

In accordance with ASOP 27, the rate of return used in the discount process is the expected rate of return taking into account the anticipated asset allocation, i.e.:

$$\text{Funding per taxpayer in Gen1} = F_1 = \frac{1}{1 + \bar{e}}$$

$$\text{Funding per taxpayer in Gen2} = F_2 = \frac{1}{1 + \bar{e}} + 1 - \frac{1 + \tilde{e}}{1 + \bar{e}}$$

where the last term in the Gen2 funding formula represents the proceeds of the Gen1 funding invested using the anticipated allocation and, since the promise is to pay \$1, the last two terms constitute the actuarial gains or losses carried from Gen1.

Since the last two terms combined with the existing fund are just sufficient to make good on the Gen1 promise, the remaining (first) term constitutes the Gen2 funding for period three. Since this is identical to the Gen1 funding for period two, it is clear that all GenK ($K > 1$) face the same situation as each other. Note further that the expected value of Gen2 funding is exactly equal to the actual Gen1 funding.

Assuming that $\alpha > 0$, it is clear that Gen2 faces some uncertainty that was not faced by Gen1. This represents a cost (price of risk) to Gen2 that Gen1 did not bear. If $\alpha = 0$, then such uncertainty will not be passed from Gen1 to Gen2 and each F_k will be identical and will equal the risk-free discount of \$1. F_1 is a decreasing function of α . Although the expected value of F_2 is also a decreasing function of α , the uncertainty of F_2 is an increasing function of α .

Note that the risk borne by Gen1 is not a function of α . In fact, there is no risk borne by Gen1. The decrease in F_1 associated with values of $\alpha > 0$ constitutes a windfall for Gen1. Similarly, any increase in α subsequent to the first period represents a windfall for the initiating generation and any decrease amounts to a payment for part of an earlier generation's windfall.

Result 3: Using an interest rate higher than the risk-free rate to compute the cost of any benefit increase constitutes a windfall for the generation that does it.

This may be seen to be an application of the principle of Result 2. With respect to the new benefit level, the granting generation enjoys the Gen1 advantage outlined above. Simply deconstruct the new benefit level of $\$(1+b)$ into \$1 of continuing benefit with respect to which GenK has no special ranking and \$b with respect to which GenK enjoys a windfall.

Note that this windfall may not belong to the taxpayer members of the granting generation. If the new benefit is exchanged for a wage concession equal to:

$$\frac{b}{1+r}$$

then the windfall belongs to the taxpayers. If, as is almost always the case in practice, the wage concession is computed as:

$$\frac{b}{1+\bar{e}}$$

then the windfall has been consumed by the employees and a new windfall will inure to future employees in every subsequent generation, assuming no subsequent adjustments in wages or benefit levels.

Result 4: The windfalls enjoyed by initiating generations come at cost to subsequent generations, a cost that is not ameliorated by equity investments.

It is tempting to believe that the windfall enjoyed by Gen1 is but the first of a series of such gains that may be enjoyed by future generations as long as the equity exposure is maintained across generations. I.e., generations who agree to diversify across time may pool their mutual risks and share the rewards. In fact, the actuarial methods we are studying herein do not achieve such sharing¹².

Consider the base case where each generation funds \$1 discounted in accordance with the riskless rate. In this base case each generation pays a certain:

¹² Intergenerational risk sharing through tax policy is possible (Smetters, 2000). Our observation is that pension actuarial methods do not implement such sharing between those taxpayers who promise pension benefits and those who must make good on the promise.

$$\frac{1}{1+r}$$

This may be compared to the case where, for $K > 1$, GenK faces an uncertain:

$$\frac{1}{1+\bar{e}} + 1 - \frac{1+\tilde{e}}{1+\bar{e}} = \frac{1+\bar{e}-\tilde{e}}{1+\bar{e}}$$

Can we determine which of these cases is preferable? Certainly the expected cost of the second case is lower than that of the base case for all $\alpha > 0$. While we might posit a risk averse utility function that would permit us to answer the preference question, the answer would turn out to be a direct result of the utility function formulation. Instead, we shall employ arbitrage pricing to determine taxpayer preferences.

In order to compute an optimal asset allocation for the investments they make during their first period of life, taxpayers desire to compute the riskless value of the obligation that they will be required to pay as taxes during their second period. But this amount is only riskless in the base case. Faced with the uncertain case, the taxpayer may elect to take a short equity position equal to $\frac{\alpha}{1+\bar{e}}$ and invest that amount in the risk-free asset.

As a result the taxpayer's effective pension funding obligation will become:

$$F_k = \frac{1+\bar{e}-r}{1+\bar{e}} = B + D$$

$$\text{where } B \equiv \frac{1}{1+\bar{e}} \text{ and } D \equiv \frac{\bar{e}-r}{1+\bar{e}}$$

What relationship does this bear to the base case? If $\alpha = 0$, it is identical to the base case. An examination of the first derivative of F_k with respect to α shows that it is strictly positive and thus F_k is a strictly increasing function of the equity exposure. In effect, our taxpayer must, after hedging the risk, be poorer than with the base case and become progressively poorer as α increases. This is not altogether surprising since Gen1's windfall is increasing in α as well.

It is useful at this point to observe the *Gen1 advantage* which may be defined relative to the base case:

$$\text{Gen1 advantage} = \frac{1}{1+\bar{e}} - \frac{1}{1+r} = -\frac{\bar{e}-r}{(1+r)(1+\bar{e})} = \frac{-D}{1+r}$$

Where the (-) sign indicates that funding has been reduced.

And similarly the risk-adjusted *GenK disadvantage*:

$$\text{GenK disadvantage} = \frac{1+\bar{e}-r}{1+\bar{e}} - \frac{1}{1+r} = \frac{(\bar{e}-r)r}{(1+\bar{e})(1+r)} = \frac{rD}{1+r}$$

which may be seen to represent the risk-free interest rate applied to the Gen1 advantage. In effect the windfall enjoyed by Gen1 constitutes an indebtedness upon which all future generations pay the interest. A “final” generation would not be able to discount its promise at any rate above the risk-free rate and thus it would have to pay the principal as well as the interest on the Gen1 advantage:

$$\text{Gen}\Omega \text{ disadvantage} = \frac{1}{1+r} + 1 - \frac{1+r}{1+\bar{e}} - \frac{1}{1+r} = D = \text{GenK disadvantage} - \text{Gen1 advantage}$$

Result 5: Spreading the economic component of actuarial gains and losses over n future periods does not alter the results above except that the first m generations may enjoy a diminishing windfall followed by $n-m$ generations who suffer an increasing burden. All generations after the n th are identical but are heavily burdened.

Having seen that mutualization of risks and rewards over time does not provide relief from the disadvantage bequeathed to future generations by Gen1, we ask whether mutualizing the resulting gains and losses by spreading over time is efficacious. All actuarial cost methods (ACMs) spread deviations from assumptions over varying forward periods. Here we consider a simple generic spreading of each year’s net gain or loss over subsequent periods in an overlapping fashion.

Consider first an example with gains and losses spread over two subsequent periods:

$$F_1 = B$$

$$F_2 = B + D/2$$

$$F_K = B + D/2 + (1+r)D/2 = B + \frac{D}{2} s_{\overline{2}|} \quad \forall K > 2$$

$$\text{Where: } s_{\overline{n}|} = \frac{(1+r)^n - 1}{r}$$

Note that the expected return and the risk-free rate are assumed to be the same from year to year¹³. Generalizing for the case where gains and losses are spread over n subsequent periods and where J is a generation greater than 1 and less than n :

$$F_1 = B$$

$$F_J = B + \frac{D}{n} s_{\overline{J}|} \quad \forall J: 1 < J \leq n$$

$$F_K = B + \frac{D}{n} s_{\overline{n}|} \quad \forall K > n$$

Since the cost of the windfall enjoyed by Gen1 is passed forward in stages, it will be the case that several (m) generations will enjoy smaller net windfall s which will be followed by a growing tab paid by the subsequent ($n-m$) generations rising to a maximum level that remains the same for all generations beginning after Gen N and prior to Gen Ω . We can solve for m :

$$B + \frac{D}{n} s_{\overline{m}|} = \frac{1}{1+r} \Rightarrow m = \frac{\log(1+nr+r)}{\log(1+r)} - 1$$

For $r = 5\%$, a 15-year spread ($n=15$) implies that m is 11.05 meaning that the first 11 generations (here taken as one year per generation) of taxpayers enjoy an advantage while the next 5 generations are increasingly burdened to a maximum burden for Gen16 and beyond. For $r = 5\%$ with $n=30$, $m=18.19$. It is clear that intergenerational wealth transfers must ultimately be exacerbated by the spreading of gains and losses over n periods. However, the string of windfalls is long enough to warm the heart of politicians and myopic taxpayers.

The Gen K disadvantage may be reformulated:

$$\frac{rD}{1+r} = D - \frac{D}{1+r}$$

¹³ More complicated, but not necessarily more illuminative, models may be developed wherein the expected value of e and the value of r vary over time

The roll-up of additional interest added to the debt during the spreading period multiplies the first term by:

$$\frac{s \frac{D}{n}}{n}$$

so that the GenK disadvantage becomes: $\frac{Ds \frac{D}{n}}{n} - \frac{D}{1+r}$

which means that the persisting disadvantage (for GenK, $K > n$) has been multiplied by:

$$\frac{(1+r)s \frac{D}{n} - n}{nr}$$

When $r=5\%$ and $n=15$, this means that the GenK disadvantage is multiplied by 10.21 and, when $n=30$, it is multiplied by 26.51. Thus, spreading does not alleviate the intergenerational transfers discussed above. Instead, it postpones the day of reckoning and multiplies the disadvantage foisted upon future generations.

Note that these numeric results depend on the procedure for spreading gains and losses over n periods. Above, the procedure assigns an increasing burden over time because the amortizations “backload” the interest charges. A more typical flat-dollar amortization might lead to somewhat smaller values of m and smaller final multipliers. The GASB-acceptable approach, spreading over future payrolls might, however, raise these values.

Result 6: The intergenerational wealth transfers detailed above do not result from the equity investment or lack thereof but arise instead from the actuarial use of an expected return in excess of the risk-free rate.

Reconsider Result 4 but assume that the base case funding is invested in the risky asset allocation. Gen1 no longer receives a windfall since the equity allocation and subsequent risk do not reduce Gen1’s cost. GenK faces an uncertain funding requirement:

$$\frac{1}{1+r} + 1 - \frac{1+\tilde{e}}{1+r} = \frac{1+r-\tilde{e}}{1+r}$$

Using the same risk adjustment as before, this may be replaced by a market-priced certainty equivalent:

$$\frac{1+r-\tilde{e}}{1+r} \text{ risk adjusted} \rightarrow \frac{1+r-r}{1+r} = \frac{1}{1+r}$$

which is identical to the base case and thus implies no wealth transfer. In effect the risks and rewards of each GenK are commensurate and each taxpayer may hedge to the risk-free base case cost.

IV Implications

Magnitude

The Public Pension Coordinating Council's 2000 Survey of State and Local Retirement Systems identifies a ratio of assets to liabilities for continuing respondents of 96%. The survey estimates that it captures 77% of the \$2.3 trillion of the assets reported by the Federal Reserve Flow of Funds (1998). Assuming that the asset liability ratio is similar for nonrespondents and respondents, we may estimate aggregate liabilities of \$2.4 trillion. The survey interest rate has remained very close to 8% for the period 1992 through 1998. Because actuaries change the assumed discount rate slowly over time, the magnitude of the generational bias varies primarily with the risk-free yield curve.

The duration of liabilities for seasoned defined benefit plans, with discount rates approximating 8%, may confidently be bracketed by ten and fifteen years depending on the relative weights of active and retired participants. The duration will be greater at lower rates of interest and thus we can approximate the \$2.4 trillion of liabilities may be revalued at between \$3 trillion and \$3.2 trillion when the risk-free rate is 6% (circa 1999) and between \$3.4 trillion and \$3.8 trillion when the risk free rate is 5% (circa 1998).

Governmental accounting and funding for pension costs

Every governmental entity would like to get as much value out of each budget dollar as it possibly can. Within the budgeting process, pensions are simply a cost and the smaller the cost, the more satisfied are the financial managers of the entity.

To this end, financial managers other than actuaries will generally cajole actuaries towards using ACM's and actuarial assumptions that reduce current costs. Because the

noneconomic assumptions are very much in the bailiwick of the actuaries and because ACM's are also an actuarial province, it is not surprising that the prime area for cajolery is in the area of the economic assumptions (discount rate and salary scale). The financial managers may argue that they, not the actuary, know better the future progression of employee wage increases. Further, because they control the investment function and have a large say in setting the asset allocation, these managers may argue that they should have great sway with respect to the assumed discount rate.

Actuaries, in general, consider themselves to be conservative¹⁴ with respect to pension funding, and they believe that they will lean towards lower discount rates and higher salary scales in order to mitigate the desires of the financial managers. Unfortunately, actuaries believe that recognizing the risk premium before the risks have been realized is not liberal¹⁵ and thus a "conservative" actuary may propose a discount rate that reflects "only" 75% of the premium above the risk-free rate. The prescriptions of ASOP 27, FASB and GASB do not provide even the most conservative actuary the support (nor an opportunity) to argue in favor of using the risk-free rate.

The use of an AAV in most funding methods makes actuaries vulnerable to a "short straddle"¹⁶ in their efforts to resist the demands of financial managers. It is not unusual, after a period in which asset returns have been better than expected, for the managers to request that the actuary perform a "market value restart"¹⁷ in order to lower current costs. If the discount rate is unchanged, costs will be lowered. Contrariwise, if environmental interest rates rise – a situation often correlated with a decline in the ratio of the AAV to market value – the request will come for an increased discount rate assumption unaccompanied by a market value restart. Although GASB 25, paragraph 36c, may comfort the actuary who wishes to resist the whipsaw of this short straddle position, the language is weak and the support fragile.

¹⁴ Trowbridge and Farr (1976) pp.27

¹⁵ Ibid.

¹⁶ A portfolio position characterized by short positions in both put and call options. In such a situation, market movement in either direction is likely to result in an option being exercised. In an insurance context, a similar situation is discussed and illustrated by Babbal (1994).

¹⁷ The AAV is reset to the current market value of assets and a new trailing average begins for future AAV's.

Historically the equity exposure in public pension plans has lagged behind that in corporate plans. In the past two decades, however, while corporate plans have maintained the equity levels reached during the 1970's, public plans have moved to narrow the gap. In effect, this means that the lesson of Result 2 has not been lost on plan managers. Each increase in the equity exposure accompanied by a liberalization of the assumed discount rate has provided a windfall. In several instances, elected officials have initiated an increase in equity exposure and have then coerced actuarial cooperation.

Benefit negotiations

"I would gladly pay you Tuesday for a hamburger, today." Wimpy (Date unknown)

The public demand for openness that surrounds the activities of democratic governments makes it necessary that the actuarial assumptions used to fund the plan and to develop the sponsor's financial statements must also be the assumptions that rule the benefit negotiation process that takes place between the financial managers of the governmental entity and its employees.

This means that these managers cannot retreat towards more conservative assumptions when they sit across the bargaining table from the representatives of the employees. As a result, public employee wages are decreased by relatively smaller amounts in exchange for generous increases in pension benefits. The systematic underpricing of the marginal cost of new benefits (Result 3) makes for bad bargains from the point of view of taxpayers. It is often the opacity of the burden placed on future taxpayers that makes both sides of the table happy with liberal actuarial assumptions.

As observed in the development of Result 3, a wage concession that is inadequate to pay fully (i.e., without risk premium) for a new benefit has an impact far beyond the initiating generation. Each new generation opens negotiations with the existing wage and benefit package as a starting point.

Occasionally, such as in New York City during the late 1970's and into the 1980's, the employee representatives will find themselves in a weak bargaining position. In such

instances, whole waves of benefit escalation may be rescinded through the adoption of less generous pension benefits for new hires. In accordance with the results presented herein, this is a substantial reversal of fortune for the taxpayers since the underpricing will also be present in this situation.

Skim funds

A common view held by professionals involved in the public plan sector is that the higher expected returns on equities constitute a reward for their astuteness in recognizing that they are “long-term investors.” Because of their understanding that they are operating in the fiduciary interests of the employees and because many of these plans include mandatory employee contributions, they believe that the higher ex-post returns on equities are a boon to be shared with employees.

It may be argued that potentially poor performance in equity returns constitutes a serious threat to the benefits promised to employees. To the extent that this is true (i.e., that taxpayers would not be called upon to make up the losses) the argument has merit. In effect, this would be an argument for using a discount rate in excess of the risk-free rate to reflect benefit insecurity when current pay is exchanged for promised benefits. To the extent, however, that taxpayers will stand behind all promises in full measure, the sharing of equity returns or any other asset results is unwarranted.

In recent periods, the returns to equity have been particularly rich and many defined benefit pension plans of all sorts have enjoyed the resulting growth in asset value. Public plan trustees often conclude that this wealth should be systematically shared between taxpayers and employees. An invention to systematize the sharing is the “skim” fund¹⁸. Retirement systems that adopt skim funds agree that “defined contribution”¹⁹ companion plans will be attached to the existing defined benefit plans and that when plan returns for one or more periods exceed an agreed upon benchmark rate, a portion of the “excess” return will be skimmed off from the defined benefit plans and will be placed in the companion plans. The assets of the companion plans will be used to fund supplementary benefits on behalf of the participants of the main plans.

¹⁸ For an example of how such a fund might be “negotiated”, see Berry (1999).

The rules for skimming can be quite complex and vary from plan to plan and from jurisdiction to jurisdiction. Nonetheless several general observations may be made:

- Skim plans constitute call options on the defined benefit plan assets written in favor of the plan participants and at the expense of the taxpayer²⁰. As a result, the net expected return to the defined benefit plan will be less than \bar{e} . When the skim takes all the return in excess of a given return k , the net expected return is given by:

$$E[\min(\tilde{e}, k)] = - (h(\alpha))^2 f(k, g(\alpha), h(\alpha)) + g(\alpha)F(k, g(\alpha), h(\alpha)) + kF(-k, g(\alpha), h(\alpha))$$

where: f is normal pdf (x, μ, σ)

F is normal cdf (x, μ, σ)

$$g(\alpha) = r + \alpha(\mu - r) = r + \alpha(\bar{q} - r) = \bar{e}$$

$$h(\alpha) = \sigma\alpha$$

When the skim takes a fraction β ($0 \leq \beta \leq 1$) of the return in excess of k :

$$E[\text{return net of skim}] = \beta E[\min(\tilde{e}, k)] + (1 - \beta)g(\alpha)$$

- Once a skim plan is in effect, employee representatives on plan investment committees have an incentive to push for ever higher equity exposure and asset volatility since this will increase the value of the call options that have been granted. This point may be subject to some mitigation when there is any sense that some large amount of volatility may threaten benefit security.
- Plan actuaries should value the skim fund benefits as options and increase plan funding requirements accordingly. Most plan actuaries are not accustomed to using the tools necessary for rigorous valuation and may use some "scenario" approaches to estimation.
- Once any skim fund is in effect, the taxpayers' risk-adjusted optimal value for α is zero. Without the skim, all asset allocations have the same risk-adjusted return, equal to the risk-free rate. Any skim with a strictly positive probability of being in the money must lower this rate. The only value of α that assures zero probability is zero.

¹⁹ The name is partly a misnomer. There are no individual accounts. These are better understood to be supplement benefit plans where the inflow of funds is defined by the skim.

²⁰ For a development of this in a Social Security context see Smetters (1999).

It is ironic that the negotiating process often finds the employee representatives demanding a skim fund in exchange for allowing the plan to increase α . This is abetted by the municipal manager's view that raising α constitutes a windfall, a view that follows from the standard actuarial practice.

- Once the skim plan is in effect there is an equity exposure (α value) that will maximize the expected return to the plan net of the skim fund²¹. Any higher α value will actually lower the expected net return. From that point onward, taxpayers will be asked to bear increased risk and receive diminished expected returns.

Pension obligation bonds

Pension Obligation Bonds (POB's) claim to be able to reduce the cash flow from a municipality in support of its pension fund by taking advantage of the assumed discount rate used by the plan's actuary – an investment bank has described its POB proposal as “arbitraging the actuary”. Anand (2/3/97) refers to “the arbitrage between what Ms. Whitman's [New Jersey Governor Christine Todd Whitman] administration anticipates paying bond purchasers and what it hopes to earn through the pension funds' investments...”

The idea is for the municipality to borrow by issuing taxable POB's (the IRS has ruled POB's cannot be tax-exempt) at its cost of borrowing, c , where $r < c < \bar{e}$, and to place the proceeds in the pension plan where the actuary will assume that they earn rate \bar{e} . Specifically, the proceeds “fund” a previously unfunded liability which was being amortized as part of the plan's annual cost.

Suppose an unfunded liability of \$1 is being amortized over n periods in level dollar amounts. The actuary would compute the amortization cost as:

$$\frac{1}{a_{\bar{n}|@ \bar{e}}} = \frac{\bar{e}}{1 - (1 + \bar{e})^{-n}}$$

²¹ For values of β greater than some β^* (where $0 < \beta^* < 1$), there will exist a finite value of α for which the net expected return is maximized (although this value of α may exceed 100%). For values of $\beta < \beta^*$, the net expected return will be strictly increasing in α .

The municipality issues an n -year POB with a \$1 principal and a self-amortizing repayment of:

$$\frac{1}{a_{\overline{n}|} @ c} = \frac{c}{1 - (1 + c)^{-n}}$$

The \$1 proceeds of the issue are placed in the plan to “satisfy” the previously unfunded liability and invested in accordance with the α -based asset mix assumed in Section III. The actuary eliminates the amortization charge from the current cost. The municipality now pays the debt service instead of the amortization charge, a “saving” to the municipality of:

$$\frac{\bar{e}}{1 - (1 + \bar{e})^{-n}} - \frac{c}{1 - (1 + c)^{-n}} > 0$$

Note that when $n = 1$, we have a case as simple as in Section III above and the above reduces to: $\bar{e} - c > 0$.

The process outlined may be divided into two processes which, taken together, have the identical financial substance as the original process:

- The pension plan locates within its portfolio a subportfolio of Treasury securities with a market value of \$1 and with cash flows exactly proportionate to the amortization schedule of the proposed POB's²². It sells this portfolio and invests the proceeds in the α -based asset mix.
- The municipality issues the POB's and places the \$1 proceeds in the pension fund where the \$1 is used to repurchase the Treasury subportfolio sold above.

This deconstruction shows that the first step is a swap or an asset reallocation whose risk-adjusted value is zero. This is as simple as the recognition that \$1 in bonds has the same value as \$1 in stocks. Their divergent expected future values are exact compensation for their differential risk. The second step constitutes a borrowing by the municipality at its borrowing rate, c , for the purpose of investing in Treasury securities

²² If, as is likely, such bonds cannot be found, many equivalent alternatives can be constructed using swaps or futures contracts. The important point here is that a large, liquid pension fund

with a rate r . With $c > r$, the differential periodic cash flow equals $\frac{c(1+c)^n}{(1+c)^n - 1} - \frac{r(1+r)^n}{(1+r)^n - 1} > 0$

which equals $c-r$ for $n=1$ and for very large values of n and is somewhat less for intermediate n . The two steps taken together make it clear that the POB process amounts to an asset reallocation that could be done independently of the bond issuance coupled with a borrowing at rate c in order to invest at rate r .

The market assigns the higher borrowing rate, c , to the municipal debt because holders of this debt face a greater risk of default or “a debt-service moratorium”²³ than do holders of Treasury debt. There are two ways to look at the debt-for-debt transaction:

- If the municipality deems its promise to the pension plan to be without risk to the plan (clearly a view somewhat at odds with the market debt rate assignments), then the transaction is simply a money loser.
- If the municipality agrees with the market that its promise is not as good as the Treasury promise, then the debt-for-debt transaction amounts to a defeasance²⁴ in favor of the plan and its participants and the net cost of the defeasance, $(c-r)$ annually, is an additional benefit to plan participants paid for by the taxpayers.

Note that, if the municipality could issue tax-exempt POB’s at a rate $c' < r$, then a true arbitrage could be effected. It is just this reasoning that led the IRS to rule that POB’s are taxable bonds.

In an unsigned editorial, *Pensions and Investments* (3/3/97), warns: “Gov. Christine Todd Whitman’s plan to issue \$2.9 billion in pension obligation bonds is good news for participants.... The state, and taxpayers should view the pension obligation bonds more cautiously...”

Earlier in the decade Los Angeles County debated the merits of POB’s. During the debate the actuarial rate remained unchanged. As the rates that would be required to

with substantially deferred cash outflows can effectively borrow at or near the Treasury borrowing rate.

²³ Mid-1970’s euphemism for default adopted by New York City.

sell the POB's rose, the proponents argued that the delays were costly to the county. "In October 1992, issuing the pension obligation bonds would have saved the county an estimated \$519 million [over twenty years]. By February 25 of this [1994] year, interest rate increases shrunk potential savings to \$318 million. By March 10, higher interest rates had reduced the estimated savings to \$240 million, according to [a plan trustee]." Hemmerick (4/18/94). This article exposes many of the political consequences arising from the use of an actuarial discount rate that is not risk-adjusted. A continuation, Hemmerick (7/25/94), shows the county bargaining position weakening through the delay and the trustees demanding additional concessions from the county.

In an exchange of Commentary (Surz, 4/4/94) and Letters (Stoufer, 5/30/94, Surz, 6/13/94), Mr. Surz, with a technical error later corrected by Mr. Stoufer, gets the substantive issues correct and concludes: "My basic premise still stands. POB's are advocated by those who benefit from them – underwriters, investment managers, consultants and beneficiaries. From whence does this benefit derive? It's paid by taxpayers, who clearly lose as they are bilked into buying off on bogus arbitrage arguments."

Writing in the Pittsburgh Post-Gazette, 1/18/98, Brian O'Neill writes: "I'm asking [a friend who knows about high-finance] about the city selling \$250 million in bonds to bail out its pension fund, asking if it's a good idea for taxpayers, when he offers eight words of solace I'll never forget: 'If we're really stupid, we're not uniquely stupid.'" An internet search indicates recent POB activities in Massachusetts, Connecticut and Georgia along with an unending river of POB's in California.

There is a simpler burlesque of the POB phenomenon that contains all the financial substance of the claimed POB advantages:

Suppose a state government issues \$1 billion of 30-year bonds promising to pay 6% interest and then takes \$500 million of the proceeds and puts it into an account where it is invested in equities. Since the expected return on the \$500 million of equities is sufficiently high to meet all of the bond payments, the state spends the remaining \$500 million immediately as it

²⁴ A special case of collateralization using Treasury securities; may be accompanied by accounting derecognition of the debt so collateralized.

pleases. An actuary says he believes that the assets are sufficient to meet the liabilities and cites ASOP 27 in support of his position.

The fundamental point of this section is this: the persistent use of expected returns in actuarial models involving risky assets ignores all of the financial economics of the last 40 years and exposes taxpayers to manipulations by those who are more sophisticated about securities markets.

V Counter-Arguments

The arguments and model of this paper may be familiar to financial economists but they represent a significant departure from the precepts of pension actuarial science. This section raises some counter-arguments from an actuarial point of view. Some of the counter arguments may be refutable from the point of view of the financial economists. Others of the arguments are not refutable and may point to areas that merit further study and consideration.

Actuaries may ask whether a refutation based in financial economics should carry any persuasive power into the actuarial discipline. The author believes that two aspects of the argument at hand bring weight to the lessons of financial economics:

- The issue argued in this paper is the setting of a discount rate assumption that varies with the anticipated asset allocation and whether or not this introduces biases into what has previously been thought to be a generationally unbiased process. This is an issue close to the center of financial economics and thus the lessons of financial economics should be carefully considered by actuaries.
- No theory is ever really proven. Science depends, however, on the concept of falsifiability. Because a theory that is falsifiable may be refuted, a falsifiable theory that is repeatedly tested and is not refuted gains credibility over time. Theories in financial economics are tested in the securities markets every day. Many theories about market behavior have been presented over the past forty years. Few remain standing after being tested in this day-by-day-by-day manner. One still standing is the theory that identical cash flows must carry the same market price. It is sometimes called “no-arbitrage pricing” or the “law of one price” (Panjer, 1998) and sometimes it is said that a security has been “priced by arbitrage” (Huang and Litzenberger,

1988). In contrast, science must remain cautious about a process such as pension valuation where thirty-year estimates are updated by new thirty-year estimates each year. It is not that the self-correcting nature of such a process is not practical, nor is it the case that the self-correction will necessarily break down. Because the process is self-correcting, however, none of the estimates are truly falsifiable. Thus, the scientific concern must be whether the year-by-year estimates can be treated as verifiable and meaningful reflections of financial reality.

Expected costs will decline over time

If defined benefit plan valuations are performed using the risk-free discount rate and the plan invests in equities, the expected plan costs will decline over time approaching the lowest level for generations $n+1$ and thereafter. In the model of this paper, there will be a decline in dollars/taxpayer. In typical applications, the decline will be in the cost as a percentage of payroll. This may be disturbing to actuaries because most ACM's are designed to create level expected costs across generations of taxpayers.

This is not a flaw in the risk-adjusted model. It is a difference in philosophy. The traditional actuarial methods favor level costs over time, deeming this to be a proper measure of intergenerational equity. A risk-adjusted model emphasizes unbiased risk-adjusted costs over time and declares this to be the measure of intergenerational equity. So why choose one or the other? Because the risk-adjusted model allows each generation to face the same ex-ante burden whereas the expected-return model provides an ex-ante reward to Gen1 that no other generation can match.

If the plan chooses not to invest in equities and uses the risk-free rate, both level expected costs and level risk-adjusted costs (and thus, intergenerational equity) will be achieved and taxpayers will hold their desired amount of risky assets in private portfolios. If the plan chooses to invest in equities and uses the risk-free rate, level risk-adjusted costs will be achieved and taxpayers may reduce their private equity exposure relative to the prior case. In this latter circumstance, taxpayers will have lower private expected wealth (ex ante, identical risk-adjusted expected wealth) offset by lower expected pension costs.

In more direct terms, it is equitable for expected costs to decline from higher levels for early generations who do not bear the equity risk to lower levels for those later generations that do.

The use of a salary scale overstates current costs and offsets understatements due to the expected-return model

This contention may well be true. Its veracity depends on a view about whether the liabilities for pensions based on future pay increases emerge when the employee provides a current year of service or when the future pay increase is realized. This paper does not take a position with respect to these varying views.

On the one hand, this paper argues that the use of expected returns on equities constitutes a wealth transfer among generations. On the other hand, it is not really the direction or magnitude of the transfer that is central to the argument. The use of the expected return is systematically biased. The fact that any other assumption or element of actuarial methodology, or any combinations thereof, may be systematically biased in such a way as to offset the expected return bias does not contradict the central argument.

A related issue follows.

The actuarial process uses many assumptions and each represents the center of a distribution of possible outcomes. What is different about the assumed discount rate that makes its use biased and these other estimates unbiased?

We might have begun the discussion of the discount rate bias with a review of utility functions and risk aversion and, in that context, might have asked whether a risk-averse agent would prefer to receive a sure return of r or an unsure return of \tilde{e} . We would have concluded that, with liquid and accessible markets, a representative risk-averse agent would be indifferent.

Consider uncertainty about mortality rates. Among 1000 men aged 65 we might expect there to be 15 deaths in one year. A risk-averse agent would prefer to exchange this uncertainty for a certainty of 15 deaths or maybe even 14 (here fewer deaths are less

“desirable” because the agent is responsible for an annuity). Does this imply that mortality assumptions also need to be risk-adjusted? If not, what is the difference between uncertain mortality and uncertain asset returns?

First, we assume that there is no significant covariance between discount and mortality rates; later, we return to the case where some uncertain liability parameters (e.g., inflation) do correlate with discount rates and asset values. Additionally we will assume that our uncertainty about mortality is limited to the realization (i.e., we believe that we have correctly estimated the distribution parameters); again, we will return to consider how misestimates of the distribution compare to the discount rate case.

The most significant distinction between discount rate and mortality uncertainty is this: we can, calling upon the Law of Large Numbers, diversify the mortality risk. If the population of the pension plan is small, the sponsor may find that it makes sense to use insurance (and the insurer may similarly use reinsurance) to achieve the necessary diversification. Note that, although there are friction costs entailed, there is no theoretical limit to the efficiency of the insurance/reinsurance process. What about diversifying the equity risk? Idiosyncratic equity risk is diversifiable in much the same way as is the mortality risk (more efficiently, perhaps). Systemic risk, however, is not and modern finance teaches that the equity risk premium is a direct consequence of the nondiversifiability of systemic risk. None of the equity premium constitutes any reward for bearing idiosyncratic risk.

What about errors in the estimation of the distribution parameters for mortality? Does this not constitute a risk that cannot be diversified? Yes, and perhaps this justifies some actuarial conservatism in projecting mortality rates. This, however, is not analogous to the systemic risk of equities. It is, rather, analogous to the misestimation of the parameters of the equity distribution. Perhaps the equity premium should be estimated as 5% rather than 6% or vice versa. The unique quality about systemic risk is that it persists even if the parameters are perfectly estimated and even after the most diligent efforts to diversify.

What about those parameters that do correlate with capital markets instruments?

Over long periods of time, inflation and salaries and thus liabilities are correlated with equity performance. Doesn't this mean that equities are less risky to a pension plan than to other investors?

To the extent that the premise is true, the arguments of this paper need to be modified. Here we address two issues: First, what is the relationship between defined benefit pension liabilities and factors found in the capital markets in places other than the yield curve? Second, what modification needs to be made to the general case presented in this paper?

This paper presumes that pension promises may be reasonably presented as fixed income streams. This is not entirely realistic, but it is a starting place. It can easily be extended to the case where post-employment COLA's are included in the defined benefit by reference to the relatively new CPI-indexed Treasury bonds.

The broader, and weaker, correlations between pension liabilities and other economic factors (various measures of inflation versus salaries, economic growth versus service termination and disability rates, productivity gains versus real salary growth) and how the equity markets react to these same factors is largely an empirical issue and beyond the scope of this paper. Nonetheless, where significant correlations may be found between the liabilities and market instruments (e.g., equities), the model herein must be modified.

Consider the case where there is a portfolio of liquid assets that perfectly tracks the liabilities of a pension plan. This portfolio may be dubbed the "liability asset". For defined benefit pension plans, the existence of such an asset might be implausible, but one can imagine a liability constructed to match available market assets. In fact this is done in many cash balance plans (Gold, 1999). Because the price of a liquid portfolio may easily and frequently be obtained, we should expect that the expected return and volatility of this portfolio may be readily estimated. In the case where the perfect liability asset exists, the model in this paper should be modified to use the expected return on this portfolio for the value of r . When, as is usually the case, there is no liability asset but there are credible correlations between equities and liabilities, the value of r should also be modified.

Consider the case where the liability-beta is β_L (assumed >0 , but the algebraic implications survive if $\beta_L \leq 0$) and there are no other non-fixed-income factors in the securities market that correlate with the liabilities. This means that there is a promise with a present value given by L and a next period value of \tilde{P} such that $\bar{P} = E[\tilde{P}] = 1$ and that \tilde{P} has the same distribution that would be obtained if $(1 - \beta_L)L$ were invested at the risk-free rate and $\beta_L L$ were invested in \tilde{q} . Thus we derive:

$$(1 - \beta_L)L(1 + r) + \beta_L L(1 + \tilde{q}) = \tilde{P}$$

$$E[(1 - \beta_L)L(1 + r) + \beta_L L(1 + \tilde{q})] = E[\tilde{P}] = 1$$

$$L = \frac{1}{1 + r'}$$

$$\text{where } r' = r + \beta_L(\bar{q} - r)$$

We assume the base case with the pension assets invested in the risk-free asset and no wealth transfers among generations. We subsequently learn that the liabilities are not riskless but that β_L applies. Now each generation is made richer by:

$$\frac{1}{1 + r} - \frac{1}{1 + r'}$$

The taxpayer who wishes to maintain her equity exposure sets aside $\frac{1}{1 + r'}$ in a β_L -

portfolio instead of $\frac{1}{1 + r}$ in a risk-free investment. Equivalently, she takes an amount

equal to $\frac{\beta_L}{1 + r'}$ out of her risk-free account and invests it in \tilde{q} . The end result, compared

to the base case is a distribution of wealth around a richer expectation with the same uncertainty as the original wealth distribution.

This result is not surprising. On a relative basis, taxpayers are richer because they have given employees a risky promise with an expected value of 1 instead of a risk-free promise of 1. The taxpayers' risk-adjusted gain is identical to the employees' risk-adjusted loss.

Mean reversion

“Any market downswing experienced in one generation will be offset by an upswing in later generations.” (Burrows, 1999)

The view above was presented by the Interim Director of the Actuarial Standards Board, Edward Burrows, in a fashion that suggests that he would expect general concurrence from his pension-actuarial audience. It is likely that he and his audience would expect the burden of proof of any contrary point of view to fall upon the expresser of the contrary opinion. Implicit in the view of most pension actuaries is a mean-reverting model of stock returns over time. The issue of whether or not stock returns exhibit mean-reverting behavior and, if so, over what periods of time, is beyond the scope of this paper.

This paper presumes a model of stock returns that is neither mean-reverting nor trend-following. While this model may not be an entirely necessary presumption, it is clear that one can construct mean-reverting models of stock market behavior that would not allow the arguments made herein.

The uncertainty that is inherent in all the actuarial assumptions other than the discount rate creates a “noisy” process. Under such circumstances, it is best to aim at the center (or slightly to the conservative side) of all the parameters taken together.

This argument is a generalization and compounding of two counter arguments above: the offsetting-assumptions argument, and the argument that equity returns are as averageable as mortality rates.

Result 4 is based upon a stationary population where the gains and losses of one generation are borne by an equal size following generation. What if growth of the population and/or general inflation allows the gains or losses to be spread over ever-larger forward generations?

Define: $f \equiv$ periodic rate of general inflation
 $p \equiv$ periodic rate of population growth
 $g \equiv (1+f)(1+p) - 1$

$$F_k = \frac{1 + \frac{\bar{e} - r}{1 + g}}{1 + \bar{e}}$$

For this risk-adjusted F_k to be no greater than $\frac{1}{1+r}$, it is necessary and sufficient that $g \geq r$. Since the risk-free rate is nominal, g is greater than f (real rate strictly positive) and thus p must be strictly greater than zero. This means that the population must be ever-increasing in order for each generation to be able to enjoy a risk and growth adjusted windfall. When the population growth rate falls below the risk-free real rate, i.e., $p < \frac{1+r}{1+f} - 1$, these Malthusian chickens come home to roost.

Even when we acknowledge that Malthus is not the final word when it comes to predicting the dangers in growth, it is still difficult to justify methodologies that transfer wealth forward and risk backward along the generation stream.

Actuarial precepts require that ACM's and assumptions remain robust in the face of populations that are winding down. This means that we do not expect a vigorous defense of existing methods and assumptions based on the growth forever premise. The recent baby-buster generation in the U.S. represents an example of the weakness in any such growth arguments. There is some irony in the observation that proponents of equity investments in the Social Security system point to public plans as their model at the time that intergenerational difficulties in the Social Security system have been highlighted by the existence of the busters.

VI Extensions

Corporate Pension Plans

This paper addresses public sector pension plans, concluding that the recognition of equity premia in the discount rate leads to intergenerational wealth transfers. The analysis may be extended into the area of single-employer pension plans sponsored by publicly traded corporations. In order to apply our arguments, we have to substitute

shareholders for taxpayers and corporations for governmental agencies. In this sector, we must make more distinctions and conclusions must be more narrowly drawn.

This paper has been able to ignore the tax-favored status enjoyed by pension trust funds primarily because public plan sponsors are generally tax exempt. In the private sector, the tax status of plans and sponsors moves close to center stage. Black (1981) and Tepper (1981) have used arbitrage arguments to question private plan investments in equities which already enjoy partial tax deferral in contrast to fixed income investments.

ASOP 27 applies to both public and private defined benefit pension plans. ERISA defines the minimum funding standards and maximum tax deductions for employer cash contributions. The Pension Benefit Guaranty Corporation insures qualified private plans when the sponsor is unable to meet its obligations. FAS 87 directs the plan sponsor's accounting for its defined benefit pension plans.

Trowbridge and Farr (1976) state that the flow of employer contributions into its pension plan serves two primary purposes:

- Budgeting of employer pension costs
- Securing employee pensions²⁵

Because the PBGC is a guarantor of much of the benefits promised to employees, Sharpe (1976) observes that the level and allocation of plan assets may be used to determine (and manage) the value of the "PBGC put". Since 1976, the PBGC has successfully lobbied Congress to increase the effective minimum funding standards and to narrow the circumstances under which the put may be exercised. Additional refinements to ERISA have narrowed the window of possible tax deductions by specifying certain discount rates to be used in the funding process. A model of the interactions between the funding rules, cash contributions, asset allocation and PBGC insurance and their impact on the benefit values from the employee and stockholder perspectives should be a worthy subject for additional research.

²⁵ Trowbridge and Farr (1976), pp. 4-5

The themes of this paper have applicability in the area of corporate accounting for defined benefit plans under the rules of FAS 87, briefly outlined in Appendix B. The arguments apply to the expected long-term returns on assets as defined by FAS 87. The definition is consistent with the ASOP 27 definition of discount rates/investment returns and is inclusive of anticipated risk premia. Thus a corporation that intends to invest more heavily in equities uses a higher long-term expected return, immediately reducing pension expense and raising reported earnings. As before, we find that an advantage (higher earnings) has been frontloaded and risks have been passed on to later accounting periods. This implicit violation of the matching principle of accounting closely parallels the wealth transfer issues in the public sector.

Whether or not this leads to an actual wealth transfer turns on the transparency of the accounting presentation of the status of the pension plan. If securities analysts are able to deduce from the disclosures that the returns are unmatched to the risk bearing, they can make adjustments (in, e.g., the price-earnings ratio). Note that it is easier to sell a security than it is to sever the relationship between a taxpayer and government.

To the extent that translucency or opacity better describes the view of the analyst, then FAS 87, by paralleling the ASOP 27 rules with respect to risk premia, can lead to transfers of wealth and risk between generations of shareholders. (Gold, 1989)

The Virtues of Smoothing

This paper is not sympathetic to the priority that various actuarial processes assign to the smooth development of pension plan costs. The high priority given to solving the sponsor's budgeting (or reporting) problem does not comport well with the financial realities of pension plans viewed as financial institutions.

The numbered results imply that smoothing is a one way street. It is not possible to take the risks realized in GenK and spread them backwards to earlier generations. As result the biases that arise from incorporating risk premia into discount rates are exacerbated by the smoothing process.

It is not necessarily the case that unbiased actuarial assumptions will lead to erratic cost development. Once the illusion that equity investments create a windfall is disposed of, plans can achieve reasonably smooth results by investing in liability-matching assets.

It remains to be investigated whether smoothing risky results forward in time can serve a purpose when there is no primary bias to exacerbate.

VII Conclusions

The actuarial assumption of discount rates that include equity risk premia is biased in favor of early generations and against later generations. This bias is not mitigated by various actuarial smoothing techniques. The use of expected returns is embedded in the actuarial psyche, where it flows somewhat naturally from seemingly similar treatment applied to such actuarial assumptions as mortality rates.

The actuaries who developed the current methods and assumptions did so at the same time that financial economics was developing an important distinction between market and nonmarket risks. Financial economics teaches that systemic market risks cannot be diversified and that this is the source of the equity risk premium. While a mean-reverting model is not explicitly argued in the actuarial literature, such a view has informed much of the thinking of actuaries practicing today. The Burrows quote at the beginning of the mean reversion subsection of the counter arguments (and its general acceptance by pension actuaries) illustrates this. If, in fact, a strong mean reversion exists in the equity markets, then the pension actuary is correct. But then, the burlesque version of pension obligation bonds is likely to prevail and government can become a perpetual money machine by borrowing to invest in equities to earn risk premia. Proponents of equity investments in the Social Security system may be engaging in the same burlesque.

We have seen that a perpetual growth scenario may also be used to postpone the intergenerational bias that arises from the unfortunate actuarial prescription for the assumed discount rate. This argument will not generally flow from actuaries, who have been trained to resist it, but it may have served to camouflage the actuarial bias to date.

Actuaries being trained today are receiving a mixed message. In their pension studies, they learn that “the present value of future contingent payments is a function of the rate of investment return, or interest at which the payments are discounted. ... The rate should represent the expected rate of return on plan assets over the long term.”²⁶ In their investment studies they are exposed to financial economics in a much more rigorous fashion than were their predecessors (Panjer, 1998). The dissonance between these has not received much attention to date and that inattention motivates this paper.

While the flaws in ASOP 27 and its predecessors may, at some future date, be addressed by actuaries better schooled in modern finance, the use of expected returns on plan assets has, over the last four decades, been implanted in statute, regulation and the practices of the accounting profession.

The implications that derive from the bias-inducing actuarially assumed discount rate should be as troubling for taxpayers and policymakers as the design flaw should be for actuaries trained in financial economics. The systematic underpricing of benefits that appeals to municipal managers at budget time has done substantial damage to these budgets in the long run because benefit improvements are negotiated and priced using the same optimism. Undervalued options, such as that exemplified by skim funds, and the highly dubious “arbitrage [of] the actuary” that leads to POB’s, injure taxpayers.

Any generation of taxpayers that acts to reduce the equity premium in its actuarial rate must injure itself by paying for the sins of its predecessors while protecting future generations. I.e., such a generation volunteers to become Gen Ω . If, however, no generation so acts, the taxpayers will continue to be shorn for the benefit of others. Because recent equity returns have been well above historic norms, it may be argued that this generation can better afford to clean the slate than most. Practically, the incorporation of equity premia in assumed rates of return could be phased out over a period of time so that the situation is corrected without one generation alone having to bear the Gen Ω burden.

²⁶ McGill et al (1996), pp. 515

In the private sector, financial analysts should be concerned with the earnings reported under FAS 87 and with the availability of the information necessary to make their own analytical adjustments.

Appendices

A Actuarial Standard of Practice No. 27, Selection of Economic Assumptions for Measuring Pension Obligations

Adopted by the ASB in December, 1996, ASOP 27 supercedes ASOP 4 with respect to economic assumptions. Generally, economic assumptions include discount rates, investment returns, inflation, compensation scales and related factors. They may be distinguished from noneconomic (demographic) assumptions relating to rates of death, disability, termination of employment, retirement, marriage, etc.

For the purposes of this paper, Section 3.6 Selecting an Investment Return Assumption and a Discount Rate is most relevant. Sections 3.5 Selecting an Inflation Assumption and Section 3.7 Selecting a Compensation Scale are pertinent to Section V Counter Arguments.

Section 3.6 states in part that “The discount rate is used to determine the present value of expected future plan payments. Generally, the appropriate discount rate is the same as the investment return assumption. But for some purposes, such as SFAS 87 [herein FAS 87] or unfunded plan valuations, the discount rate may be selected independently of the plan’s investment return assumption, if any.”

Section 3.62 Constructing the Investment Return Range details two example methods:

- a. Building-Block Method – after identifying asset classes and, for each such class, estimating real returns and inflation, “compute an average, weighted real-return range reflecting the plan’s expected asset class mix; and ... combine [this] ... with the expected inflation range.”
- b. Cash Flow Matching Method – “Under the cash flow matching method, the expected future investment return range is viewed as the combination of (i) the

internal rate of return on a bond portfolio [of noncallable, high-quality corporate or U.S. government bonds] with interest and principal payments approximately matching the plan's expected disbursements, and (ii) a risk adjustment range." The risk adjustment range should reflect "expected future plan investments in equities or other asset classes besides high-quality bonds."

Clearly, the prescription given by ASOP 27 is that the equity premium (or other risk premia) is to be included in the discount rate when the plan is anticipated to invest in equities and other risky assets. In the present environment, the equity risk premium amounts to several percent per annum and equity allocations frequently represent a majority or near majority of plan assets. Thus, for such plans today, ASOP 27 holds that use of the risk-free rate is outside of the actuarial standard of practice.

B Statement No. 87 of the Financial Accounting Standards Board (FAS 87), Accounting for Pensions by Employers

FAS 87 provides the standards of accounting for private employers who sponsor defined benefit pension plans. The priority of pension accounting is to recognize pension cost over the service period of each employee covered by the plan. This objective leads, in part, to the choice of an accrued benefit actuarial cost method since such methods do not spread costs over aggregations of employees. The chosen method is a modification of an earlier actuarial method often identified as Projected Unit Credit (PUC).

The Projected Unit Credit method assigns to each year of service for each employee a pro rata share of the benefit that will have been earned upon separation from service, reflecting an estimate of any past or future wages that may be taken into account by the defined benefit formula. If, for example, an actuary assumes that an employee, hired at age 35 will retire at age 65 and will have a pay history at age 65 that entitles him to an annual retirement benefit of \$30,000, the method will dictate that \$1000 of that benefit be allocated to each of the thirty years of service.

If that employee is now 50 years of age, the method will recognize \$15,000 of the projected \$30,000 benefit as having been accrued to date and will recognize an additional \$1000 to be earned in the current accounting year. The \$15,000 benefit,

multiplied by an appropriate annuity factor and discounted for the period prior to the commencement of the benefit, becomes the Projected Benefit Obligation (PBO) attributable to this employee. The PBO for the plan will be the sum for all current participants of the plan including the remaining annuity value for all those no longer actively employed. The \$1000 of benefit to be allocated to the current period will also be multiplied and discounted by the same factors. The resulting value is called the Service Cost (SC) for this employee and will be aggregated across employees to yield the plan's Service Cost for this period. Using the language of traditional actuarial writings, the PBO would be called the Accrued Liability under the PUC method. The SC would be called the Normal Cost (NC) under the PUC method.

Under the traditional PUC method, the annual cost for the plan would then be determined as: $NC + \text{Amortizations}$, where the amortizations would be a periodic payment whose present value would equal the difference: $PBO - AAV$ which would be called the Unfunded Accrued Liability. The amortization amount for the current period would depend upon the history of events that created the divergence between the AAV and the PBO. Such items as an existing difference at the commencement of the plan, or a difference created by changes to the benefit formula of the plan or to changes in actuarial assumptions, or to differences over time between actual and assumed experience (actuarial gain or loss) would be amortized over various fixed periods.

FAS 87 modifies this method in several ways:

- The amortizations in the traditional method include interest on the unfunded accrued liability in each amortization element. FAS 87 separates the interest component and applies it to the PBO and the AAV (renamed the Market-Related-Value, MRV) separately. The unfunded accrued liability arising from the initial application of FAS 87 and any changes therein attributable to changes in the plan formula or actuarial assumptions are spread (amortized) over time without interest. The gain or loss is accumulated (the asset component of gain and loss is only included herein after it has entered into the MRV) and when it is sufficiently large (it exceeds a optional buffer zone known as a "corridor") it is spread over time without interest.

- The interest component applied to the PBO is computed using a discount rate identified as the “settlement rate”. This same rate, designated i below is also used to compute the PBO itself and the SC. It is determined at each valuation date based on the returns available on high-quality fixed income securities or annuities. In the sense that it contains no equity risk premium and does not take the plan’s asset allocation into account, it is conceptually similar to the risk-free rate used elsewhere in this paper. ASOP 27 identifies this as a “prescribed rate” and exempts it from the general ASOP 27 rules pertaining to the selection of an investment return range.
- The interest component applied to the MRV is computed using an expected long-term rate of return on plan assets that is consistent with the ASOP 27 rules. This rate designated j below is generally left unchanged for several years.

The PUC formula may then be reconstituted as: $iPBO + SC + \text{Amortizations} - jMRV$. Herein, amortizations are without interest. Amortization of accumulated gains or losses only considers such gains or losses outside of the corridor. The asset component of gains and losses only recognizes the difference in actual versus expected return to the extent that the difference has been filtered through the averaging process used to develop the MRV. As in most ACM’s, the special treatment of the divergence between asset assumptions and experience amounts to an extra degree of smoothing.

C Statement No. 25 of the Governmental Accounting Standards Board (GASB 25), Financial Reporting for Defined Benefit Pension Plans and Note Disclosures for Defined Contribution Plans

This statement outlines the standard methods and provides guidance with respect to assumptions used by actuaries and accountants in the preparation of financial reports for public (governmental) employee pension plans. Of most interest with respect to the subject of this paper is Paragraph 36, subparagraphs b-f.

- Subparagraph b provides linkage to ASOP 4 and its successors including ASOP 27 which is a successor to parts of ASOP 4.
- Subparagraph c says, in part, “the **investment return assumption (discount rate)** should be based on an estimated long-term investment yield for the plan with

consideration given to the nature and mix of current and expected plan investments and the basis used to determine the **actuarial value of assets** (paragraph 36e).”

- Subparagraph d names acceptable **actuarial cost methods** including “**entry age, frozen entry age, attained age, frozen attained age, projected unit credit**, or the aggregate actuarial cost method...”
- Subparagraph e describes **Actuarial Asset Value** methodology and indicates that it is to be **market related**.
- Subparagraph f details amortizations required for use with some of the actuarial cost methods. amortizations may be made as fixed dollars over periods ranging from 10 to 40 years or as a **percentage of the projected payroll** of active plan members..

Bolding above follows that of the document and highlights defined terms. Paragraph 45 indicates that the various actuarial cost methods (ACMs) are as defined in ASOP 4.

D Statement No. 27 of the Governmental Accounting Standards Board (GASB 27), Accounting for Pensions by State and Local Government Employers

This document is the companion to GASB 25 prescribing the inclusion of pension-related financial information on the books of account of the employing entities.

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