

# Financial Engineering

## Implications for Pension Actuaries

# A Puzzle

- Non-dividend-paying publicly-traded stock, currently \$100/share.
- Party A contracts to pay B exactly \$1 when the stock first hits \$150 per share.
- What price should B pay A now?

# Puzzle

- Make and state any assumptions you wish (e.g., risk-free rate, equity risk premium, standard deviation of the stock's returns)
- May use standard instruments, e.g., shares of stock, risk-free bills, standard options.

# Discount Approach

- Assume, e.g.: risk-free rate 5%, expected stock return 10%, standard deviation 20%.
- Compute probability  $P(t)$  that stock first hits \$150 per share at time  $t$ .
- Integrate:  $\int_t P(t) \exp[-t \log(1+i)]$

# Discount Approach

- But what rate for  $i$ ?
- This contract is risky, implies  $>5\%$ .
- This contract is leveraged, so  $>10\%$ ? How much greater?
- Hmm. Are we all sure that the answer depends on the variables we have defined?

# Hedge Approach

- What asset, combination of assets or what trading strategy can we use to replicate the contract?

# Hedge Approach

- What asset, combination of assets or what trading strategy will be worth exactly \$1 when the stock first hits \$150?

# Hedge Approach

- What asset will be worth exactly \$1 when one share of this stock is worth exactly \$150?



# Hedge Approach

- Aha!  $1/150$  of a share will be worth \$1.
- What is current price of  $1/150$  of a share?
- $66\frac{2}{3}$  cents
- Is that right? No matter the risk or the expected return or the risk-free rate?
- What if the stock never hits \$150?

# Frameworks

- Discount approach - the traditional actuarial and accounting standard.
- Hedge approach - the modern financial approach. (aka “pricing by arbitrage”, “no arbitrage pricing”)

# A Little History

- 1958 - Modigliani-Miller use replication (hedge) approach.
- 1968 - Hamilton-Jackson itemize actuarial asset valuation methods.
- 1960's - economists try to value options using discount - cannot determine  $i$ .

# A Little History

- 1972 - Black-Scholes - hedge approach => option value = hedge portfolio price.
- Black (1980), Tepper (1981) show that DB should invest only in taxable fixed income.
- Actuarial techniques favor equity investing. Violation of no-arbitrage framework.

# What's Wrong with That?

- Suppose a \$.75 price for the A:B contract.
- We sell lots of such contracts. Profitably replicate contract for  $66\frac{2}{3}$  cents.
- Violate no-arbitrage price, invite arbitrage.
- DB actuaries always violate no-arbitrage, inviting arbitrage with perverse effects.
- Examples:

# DB Arbitrage Examples

- POB's (see Risk and Rewards, Sept., 2000).
- Corporate POB's.
- Dedicated bond portfolios, circa 1982
- NationsBank: DC ---> DB
- Actuarial pricing used for plan amendments and negotiations.
- ASOP 27 must be amended.