(2.) $3 \cot x + \sqrt{3} = 0$ here is the problem

$-\sqrt{3} \quad -\sqrt{3}$ subtract $\sqrt{3}$ from each side

$3 \cot x = -\sqrt{3}$ subtract

$\frac{3}{3} \quad \frac{-\sqrt{3}}{3}$ divide each side by 3

$\cot x = -\sqrt{3}/3$ cancel

$\cot x = -3/\sqrt{3}$ multiply top and bottom by $\sqrt{3}$

$\frac{\cos x}{\sin x} = -\sqrt{3}$ divide and use cos and sin

$\frac{\cos x}{\sin x} = \frac{-\sqrt{3}/2}{1/2}$ divide top and bottom by 2

$x = 5\pi/6$ use the unit circle

(3.) $(\sin x - 1)(2 \sin x + 1) = 0$ here is the problem

$2 \sin x + 1 = 0$ $\sin x - 1 = 0$ set ea factor = to 0

$-1 \quad -1$ $+1 \quad +1$ add this to ea side

$2 \sin x = -1$; $\sin x = 1$ add

$\frac{2}{2} \quad \frac{-1}{2}$ divide ea side by 2

$\sin x = -1/2$; $\sin x = 1$ cancel

results: $x = 7\pi/6$; $x = 11\pi/6$; $x = \pi/2$

(4.) $(2 \cos x + 1)(\cos x - 1) = 0$ here is the problem

$2 \cos x + 1 = 0$ $\cos x - 1 = 0$ set each factor = to 0

$-1 \quad -1$ $+1 \quad +1$ add this to each side

2 cos x = -1 ; cos x = 1 add

$\frac{2}{2}$ $\frac{}{2}$ divide each side by 2

cos x = -1/2 ; cos x = 1 cancel

x = 2pi/3 x = 5pi/3 x = 0

(5.) $3\tan^2 x - \sqrt{3} \tan x = 0$ here is the problem

(tan x)(3 tanx - √3) = 0 factor

tan x = 0 3 tan x - √3 = 0 set each factor equal to 0

\+ √3 +√3 add √3 to each side

tan x = 0 ; 3 tan x = √3 add

$\frac{}{3}$ $\frac{}{3}$ divide each side by this

tan x = 0 ; tan x = √3/3 cancel

x = 0 ; x = pi ; x = pi/6 ; x = 5pi/6 use the unit circle

(6.) $\sec^2 x + 2 \sec x = 0$ here is the problem

(sec x)(sec x + 2) = 0 factor

sec x + 2 = 0 set this factor equal to 0

-2 -2 subtract 2 from each side

sec x = -2 subtract

cos x = -1/2 reciprocals

x = 2pi/3 x = 4pi/3

(9.) $\sqrt{3}\cot x + 1 = 0$ here is the problem

-1 -1 subtract 1 from each side

$\sqrt{3}\cot x = -1$ subtract

$\cot x = -1/\sqrt{3}$ divide each side by this

x = 2pi/3 x = 5pi/3 use the unit circle

(10.) $2\sin x + \sqrt{3} = 0$ here is the problem

$\sin x = -\sqrt{3}/2$ solve for sin x

x = 4pi/3 x = 5pi/3 use the unit circle

(11.) $2\sin^2 x - \sin x - 1 = 0$ here is the problem

$(2\sin x + 1)(\sin x - 1) = 0$ factor

$2\sin x + 1 = 0$ $\sin x - 1 = 0$ set each factor = to 0

-1 -1 +1 +1 add this to each side

$2\sin x = -1$; $\sin x = 1$ add

$\frac{2\sin x}{2} = \frac{-1}{2}$; $\frac{\sin x}{1} = \frac{1}{1}$ divide each side by 2

$\sin x = -1/2$; $\sin x = 1$ cancel

$x = 7\pi/6$; $x = 11\pi/6$; $x = \pi/2$

(12.) $2\tan^2 x - 3\sec x + 3 = 0$ here is the problem

$2\sin^2 x - 3\cos x + 3\cos^2 x = 0$ multiply thru by $\cos^2 x$

$2(1 - \cos^2 x) - 3\cos x + 3\cos^2 x = 0$ pythagorean id

$2 - 2\cos^2 x - 3\cos x + 3\cos^2 x = 0$ multiply thru

$\cos^2 x - 3\cos x + 2 = 0$ combine like terms

$(\cos x - 2)(\cos x - 1) = 0$ factor

$\cos x - 1 = 0$ set this factor equal to 0

$+1 \quad +1$ add 1 to each side

$\cos x = 1$ add

$x = 0 \quad x = 2\pi$

(15.) $3\sin^2 x - \cos^2 x = 0$ here is the problem

$3(1 - \cos^2 x) - \cos^2 x = 0$ pythagorean id

$3 - 3\cos^2 x - \cos^2 x = 0$ multiply thru

$-4\cos^2 x + 3 = 0$ combine like terms

$4\cos^2 x - 3 = 0$ multiply thru by -1

$+3 \quad +3$ add 3 to each side

$4\cos^2 x = 3$ add

$\frac{4}{4} \qquad \frac{3}{4}$ divide each side by 4

$\cos^2 x = 3/4$ cancel

$\cos x = \sqrt{3}/2 \quad \cos x = -\sqrt{3}/2$ take square roots

$x = \pi/6 \quad x = 11\pi/6 \quad x = 5\pi/6 \quad x = 7\pi/6$

(16.) $\sqrt{3}\csc^2 x + 2\csc x = 0$ here is the problem

$\sqrt{3} + 2\sin x = 0$ multiply thru by $\sin^2 x$ and cancel

$-\sqrt{3} \quad -\sqrt{3}$ subtract $\sqrt{3}$ from each side

$2 \sin x = -\sqrt{3}$ subtract

$\sin x = -\sqrt{3}/2$ divide ea side by 2, cancel

$x = 4\pi/3 \quad x = 5\pi/3$ use the unit circle

(17.) $\sin x = \cos x$ here is the problem

$x = \pi/4 \quad x = 5\pi/4$ use the unit circle

(18.) $\cos x = 3 \cos x - 2$ here is the problem

$2 = 2 \cos x$ add 2 to each side, subtract cos x from each side

$1 = \cos x$ divide each side by 2, cancel

$x = 0 \quad x = 2\pi$

(19.) $4 \sin^2 x - 4 \sin x + 1 = 0$ here is the problem

$(2 \sin x - 1)(2 \sin x - 1) = 0$ factor

$2 \sin x - 1 = 0$ set this factor equal to 0

$+ 1 \quad +1$ add 1 to each side

$2 \sin x = 1$ add

$\frac{2}{2} \quad \frac{1}{2}$ divide each side by 2

$\sin x = 1/2$ cancel

$x = \pi/6 \quad x = 5\pi/6$ use the unit circle

(20.) $\frac{(\sin x)}{1 + \cos x} = 1$ here is the problem

$\sin x = 1 + \cos x$ multiply each side by 1 + cos x

$x = \pi/2$ $x = \pi$ use the unit circle

(21.) $\cos 2x + \sin x = 1$ here is the problem

$\cos^2 x - \sin^2 x + \sin x = 1$ double angle id for cos

$(1 - \sin^2 x) - \sin^2 x + \sin x = 1$ pythagorean identity

$1 - 2\sin^2 x + \sin x = 0$ combine like terms

$2\sin^2 x - \sin x - 1 = 0$ multiply thru by -1

$(2 \sin x + 1)(\sin x - 1) = 0$ factor

$2 \sin x + 1 = 0$ $\sin x - 1 = 0$ set ea factor = to 0

$-1 \quad -1$ $+1 \quad +1$ add this to ea side

$2 \sin x = -1$; $\sin x = 1$ add

$\sin x = -1/2$ $\sin x = 1$ div ea side by 2

$x = 7\pi/6$ $x = 11\pi/6$ $x = \pi/2$ use the unit circle

(22.) $\sin 2x + \cos x = 0$ here is the problem

$2 \sin x \cos x + \cos x = 0$ double angle id for sin

$(\cos x)(2 \sin x + 1) = 0$ factor

$\cos x = 0$ $2 \sin x + 1 = 0$ set ea factor = to 0

$-1 \quad -1$ subt 1 fr ea side

$\cos x = 0$; $2 \sin x = -1$ subt

$\cos x = 0$; $\sin x = -1/2$ divide ea side by 2

x = pi/2 ; x = 3pi/2 ; x = 7pi/6 ; x = 11pi/6

[use the unit circle]

(23.) 4 tan x + sin 2x = 0 here is the problem

4 tan x + 2 sin x cos x = 0 double angle id for sin

(sin x)(4 sec x + 2 cos x) = 0 facto

(sin x)(4 + 2 $\cos^2$ x) = 0 multiply thru by cos x, cancel

sin x = 0 4 + 2 $\cos^2$ x = 0 set ea factor = to 0

-4 -4 subt 4 fr ea side

sin x = 0 ; 2 $\cos^2$ x = -4 subtract

x = 0 x = pi x = 2pi

(24.) sin 2x = 2 sin x here is the problem

2 sin x cos x = 2 sin x double angle id for sin

- 2 sin x - 2 sin x subt this fr ea side

2 sin x cos x - 2 sin x = 0 subt

(2 sin x)(cos x - 1) = 0 factor

sin x = 0 cos x - 1 = 0 set ea factor = to 0

+1 +1 add 1 to each side

$\sin x = 0$; $\cos x = 1$ add

$x = 0$; $x = \pi$

(25.) $\tan 2x \cot x - 3 = 0$ here is the problem

$$\frac{\sin 2x \cos x}{\cos 2x \sin x} - 3 = 0$$ write as sin/cos

$$\frac{2 \sin x \cos x \cos x}{(1 - 2\sin^2 x)(\sin x)} - 3 = 0$$ double angle id's

$$\frac{2 \cos^2 x}{1 - 2\sin^2 x} - 3 = 0$$ cancel

$2 \cos^2 x - 3(1 - 2 \sin^2 x) = 0$ multiply thru by this

$2(1 - \sin^2 x) - 3(1 - 2 \sin^2 x) = 0$ pythagorean id

$2 - 2 \sin^2 x - 3 + 6 \sin^2 x = 0$ multiply thru

$4 \sin^2 x - 1 = 0$ combine like terms

$+ 1 \quad +1$ add 5 to each side

$4 \sin^2 x = 1$ add

$\frac{}{4} \quad \frac{}{4}$ divide ea side by 4

$\sin^2 x = 1/4$ cancel

$\sin x = 1/2$ $\sin x = -1/2$ take sq roots

$x = \pi/6$ $x = 5\pi/7$ $x = 7\pi/6$ $x = 11\pi/6$

[use the unit circle]

(26.) $\cos^2 x + \cos 2x = 5/4$ here is the problem

$\cos^2 x + \cos^2 x - \sin^2 x = 5/4$ double angle id

$\cos^2 x + \cos^2 x - 1 + \cos^2 x = 5/4$ pythagorean id

$3 \cos^2 x - 1 = 5/4$ combine like terms

$+ 1 + 4/4$ add 1 to ea side

$3 \cos^2 x = 9/4$ add

$\cos^2 x = 3/4$ multiply ea side by 1/3, cancel

$\cos x = \sqrt{3}/2$ $\cos x = -\sqrt{3}/2$ take sq roots

c = pi/6 x = 11pi/6 x = 5pi/6 x = 7pi/6

[use the unit circle]

(27.) $\sin [(pi/4) + x] - \sin [(pi/4) - x] = \sqrt{2}/2$

$\sin (pi/4) \cos x + \cos (pi/4) \sin x$

$- [\sin (pi/4) \cos x - \cos (pi/4) \sin x] = \sqrt{2}/2$

$2 \cos (pi/4) \sin x = \sqrt{2}/2$ combine like terms

$2(\sqrt{2}/2) \sin x = \sqrt{2}/2$ use the unit circle

$\sqrt{2} \sin x = \sqrt{2}/2$ cancel

$\sin x = 1/2$ divide ea side by $\sqrt{2}$

x = pi/6 x = 5pi/6 use the unit circle

(28.) $\cos [(pi/4) + x] + \cos [(pi/4) - x] = 1$

$\cos (pi/4) \cos x - \sin (pi/4) \sin x$

+ cos (pi/4) cos x + sin (pi/4) sin x = 1 double angle id

2 cos (pi/4) cos x = 1 combine like terms

2(√2/2) cos x = 1 use the unit circle

√2 cos x = 1 cancel

cos x = 1/√2 divide ea side by √2

x = pi/4 x = 7pi/4 use the unit circle

(33.) cos 2x + 3 cos x - 1 = 0 here is the problem

$2\cos^2$ - 1 + 3 cos x - 1 = 0 double angle id

$2\cos^2$ x + 3 cos x - 2 = 0 combine like terms

(2 cos x - 1)(cos x + 2) = 0 factor

2 cos x - 1 = 0 set this factor equal to 0

\+ 1 +1 add 1 to each side

2 cos x = 1 add

cos x = 1/2 divide each side by 2, cancel

x = pi/3 x = 5pi/3 use the unit circle

(34.) |sin x| = 1/2 here is the problem

sin x = 1/2 sin x = -1/2 property of absolute value

x = pi/6 x = 5pi/6 x = 7pi/6 x = 11pi/6

[use the unit circle]

(35.) sin 3x + sin x = 0 here is the problem

$\sin(2x + x) + \sin x = 0$ write 3x as 2x + x

$\sin 2x \cos x + \cos 2x \sin x + \sin x = 0$ double angle id

$2 \sin x \cos x \cos x + (1 - 2\sin^2 x)(\sin x) + \sin x = 0$

$2 \sin x \cos^2 x + \sin x - 2 \sin^3 x + \sin x = 0$ multiply thru

$2 \sin x \cos^2 x + 2 \sin x - 2 \sin^3 x = 0$

[combine like terms]

$2 \cos^2 x + 2 - 2 \sin^2 x = 0$ divide thru by sin x, cancel

$\cos^2 x + 1 - \sin^2 x = 0$ divide thru by 2, cancel

$2\cos^2 x = 0$ pythagorean id

$\cos^2 x = 0$ divide thru by 2, cancel

$\cos x = 0$ take the sq root of each side

(36.) $6\cos^2 x + 5 \cos x + 1 = 0$

$(3 \cos x + 1)(2 \cos x + 1)$ factor

$3 \cos x + 1 = 0$ $2 \cos x + 1 = 0$ set ea factor = to 0

$-1 \quad -1$ $-1 \quad -1$ subt 1 from each side

$3 \cos x = -1$; $2 \cos x = -1$ subt

$\cos x = -1/3$; $\cos x = -1/2$ divide ea side by 3 and 2

and cancel

$x = \arccos(-1/3)$; $x = 2\pi/3$ $x = 4\pi/3$

(37.) $2 \tan x - 2 \cot x = -3$ here is the problem

$+ \quad 3 \quad + \quad 3$ add 3 to each side

$2 \tan x - 2 \cot x + 3 = 0$ add

$2 \tan^2 x - 2 + 3 \tan x = 0$ multiply thru by tan x

$(2 \tan x - 1)(\tan x + 2) = 0$ factor

$2 \tan x - 1 = 0$; $\tan x + 2 = 0$ set ea factor = to 0

$+1 \quad +1$; $- 2 \quad -2$ add this to ea side

$2 \tan x = 1$; $\tan x = -2$ add

$\tan x = 1/2$; $\tan x = -2$ div ea side by 2, cancel

$x = \arctan (1/2)$; $x = \arctan (-2)$

(38.) $2 \sin^3 x - \sin x = 0$ here is the problem

$(\sin x)(2 \sin^2 x - 1) = 0$ factor

$2 \sin^2 x - 1 = 0$ $\sin x = 0$

$- \cos 2x = 0$ $\sin x = 0$ double angle id for cos

$2x = \pi/2$ $2x = 3\pi/2$ $2x = 5\pi/2$ $2x = 7\pi/2$

[use the unit circle]

$x = \pi/4$; $x = 3\pi/4$; $x = 5\pi/4$; $x = 7\pi/4$

$x = 0$

[div ea side by 2, cancel]

(39.) $\sin 2x + \cot 3x = 0$ here is the problem

$$\sin 2x + \frac{\cos(2x + x)}{\sin(2x + x)} = 0 \qquad \text{write } 3x \text{ as } 2x + x$$

$$\sin 2x + \frac{\cos 2x \cos x - \sin 2x \sin x}{\sin 2x \cos x + \cos 2x \sin x} = 0$$

$$\sin 2x + \frac{(1 - 2\sin^2 x)(\cos x) - 2 \sin x \cos x \sin x}{2\sin x \cos x \cos x + (2\cos^2 x - 1)(\sin x)} = 0$$

[double angle id's]

$$2 \sin x \cos x + \frac{\cos x - 2 \sin^2 x \cos x - 2 \sin^2 x \cos x}{2 \sin x \cos^2 x + 2\sin x \cos^2 x - \sin x} = 0$$

[multiply thru parenthses]

$$2 \sin x \cos x + \frac{\cos x - 4 \sin^2 \cos x}{4 \sin x \cos^2 x - \sin x} = 0 \qquad \text{combine like terms}$$

$$2 \sin x \cos x + \frac{(\cos x)(1 - 4 \sin^2 x)}{(\sin x)(4\cos^2 x - 1)} = 0 \qquad \text{factor}$$

$$2(\sin^2 x)(\cos x)(4 \cos^2 x - 1) + (\cos x)(1 - 4\sin^2 x) = 0$$

[multiply thru by $(\sin x)(4\cos^2 x - 1)$ and cancel]

$$-8(\sin^4 x)(\cos x) + (\cos x)(1 - 2 \sin x)(1 + 2 \sin x) = 0$$

[simplify]

$$(\cos x)[-8\sin^4 x + (1 - 2 \sin x)(1 + 2 \sin x)] = 0$$

$$(\cos x)[8\sin^4 x + 4 \sin^2 x - 1] = 0 \qquad \text{multiply}$$

$b^2 - 4ac$ use the discriminant formula

$= (4)^2 - 4(8)(-1)$ make substitutions

$= 48$ multiply combine like terms

$\sin^2 x = [-b + \sqrt{b^2 - 4ac}]/(2a)$ use the quadratic formula

$\sin^2 x = [-4 + \sqrt{(4)^2 - 4(8)(-1)}]/(2*8)$ make substitutions

$\sin^2 x = [-4 + \sqrt{48}]/(16)$ multiply add

$\sin^2 x = [-1 + \sqrt{3}]/4$ divide thru by 4, cancel

$x = 0.442$ radians use calculator

$\cos x = 0$ set this factor equal to 0

$x = \pi/2$ $x = 3\pi/2$

results: $x = 0.442$; $x = \pi/2$, $x = 3\pi/2$

(40.) $4 \sin^4 x + \sin^2 x = 3$ here is the problem

$- 3 \quad -3$ subt 3 fr ea side

$4 \sin^4 x + \sin^2 x - 3 = 0$ subtract

$(4 \sin^2 x - 3)(\sin^2 x + 1) = 0$ factor

$4 \sin^2 x - 3 = 0$ set this factor = to 0

$+ \ 3 \ +3$ add 3 to each side

$4 \sin^2 x = 3$ add

$\sin^2 x = 3/4$ divide each side by 4, cancel

sin x = √3/2 sin x = -√3/2 take sq roots

x = pi/3 x = 2pi/3 x = 4pi/3 x = 5pi/3

[use the unit circle]