Solve the following triangles:

(1.)   a = 8; c = 7;   B = 135   here is the problem

   $b^2 = a^2 + c^2 - 2ac \cos B$ use the law of cosines

   $b^2 = (8)^2 + (7)^2 - 2(8)(7)\cos 135$ make substitutions

   b = 14   use calculator

   $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$ use the law of sines

   $\dfrac{\sin A}{8} = \dfrac{\sin 135}{14}$ make substitutions

   14 sin A = 8 sin 135   cross multiply

   $\dfrac{}{14}$ $\dfrac{}{14}$ divide each side by 14

      sin A = (8 sin 135)/(14)   cancel

    A = arcsin [(8 sin 135)/(14)]   take arcsin of each side

    A = 24   use calculator

    A + B + C = 180   use the triangle sum theorem

   24 + 135 + C = 180   make substitutions

    C + 159 = 180   combine like terms

     - 159 -159   subtract 159 from each side
  _____
     C   = 21   subtract

result: A = 24 ;   B = 135 ;   C = 21

         a = 8 ; b = 14 ;   c = 7

(2.)   a = 30;   b = 50 ;   C = 25   here is the problem

$c^2 = a^2 + b^2 - 2ab \cos C$ use the law of cosines

$c^2 = (30)^2 + (50)^2 - 2(30)(50)\cos 25$ make substitutions

c = 26   use calculator

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$ use the law of sines

$$\frac{\sin A}{30} = \frac{\sin 25}{26}$$ make substitutions

$\dfrac{26 \sin A}{26} = \dfrac{30 \sin 25}{26}$ cross multiply   divide each side by 26

sin A = (30 sin 25)/(26)   cancel

A = arcsin [(30 sin 25)/(26)]   take arcsin of each side

A = 29   use calculator

A + B + C = 180

29 + B + 25 = 180   make substitutions

B + 54 = 180   ;   combine like terms

$\begin{array}{r} -54 \quad -54 \\ \hline B = \quad 126 \quad ; \end{array}$ subtract from each side

          subtract

results:

A = 29 ; B = 126 ;   C = 25

a = 30; b = 50 ; c = 26

(3.) a = 17; b = 23 ;   c = 32   here is the problem

   $c^2 = a^2 + b^2 - 2ab \cos C$ use the law of cosines

   $32^2 = (17)^2 + (23)^2 - 2(17)(23)\cos C$ make substitutions

   $1024 = 818 - 782 \cos C$ use calculator

-1024 = -818 + 782 cos C   multiply thru by -1

 +818   +818   add 818 to each side
_____
 -206   =   782 cos C   add

 ____   _____
 782   782   divide each side by 782

 cos C = -206/782   cancel

C = arccos (-206/782)   take the arccos of each side

C = 105   use calculator

  sin C   sin A
  _____ = _____
   c   a   use the law of sines

  sin 105   sin A
_____ = _____
  32   17   make substitutions

 32 sin A = 17 sin 105   cross multiply
_____   _____
    32   32   divide each side by 32

 sin A = (17 sin 105)/(32)   cancel

A = arcsin [(17 sin 105)/(32)]   take arcsin of ea side

A = 31   use calculator

A + B + C = 180   use the triangle sum theorem

31 + B + 105 = 180   make substitutions

B + 136 = 180   combine like terms

-136   -136   subtract 136 from each side

_____

B =   44   subtract

results:

A = 31;   B = 44 ;   C = 105

a = 17;   b = 23 ;   c = 32

(4.)   A = 52;   B = 28;   c = 87   here is the problem

A + B + C = 180   use the triangle sum theorem

52 + 28 + C = 180   make substitutions

C + 80 = 180   combine like terms

- 80   -80   subtract 80 from each side

_____

C = 100   subtract

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$ use the law of sines

$$\frac{\sin 52}{a} = \frac{\sin 100}{87}$$ make substitutions

a sin 100 = 87 sin 52   cross multiply

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  sin 100   sin 100   divide each side by sin 100

a = (87 sin 52)/(sin 100)   cancel

a = 70   use calculator

$b^2 = a^2 + c^2 - 2ac \cos B$ use the law of cosines

$b^2 = (70)^2 + (87)^2 - 2(70)(87)\cos 28$ make substitutions

b = 41.4   use calculator

results: A = 52 ;   B = 28;   C = 100

         a = 70 ;   b = 41.4 ;   c = 87

(5.)   a = 9; b = 6 ;   c = 5

    $c^2 = a^2 + b^2 - 2ab \cos C$ use the law of cosines

    $5^2 = 9^2 + 6^2 - 2(9)(6)\cos C$ make substitutions

    25 = 117 - 108cos C   multiply and ad

 -25 = -117 + 108cos C   multiply thru by -1

 + 117 +117   add 117 to each side

————————————————————————

   92 =   108cos C   add

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  108   108   divide each side by 108

 cos C = (92/108)   cancel

  C = arccos (92/108)   take arccos of each side

  C = 31.5   use calculator

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$ use the law of sines

$$\frac{\sin A}{9} = \frac{\sin 31.5}{5}$$ make substitutions

5 sin A = 9 sin 31.5   cross multiply

$\frac{}{5} \quad \frac{}{5}$ divide each side by 5

sin A = (9/5)sin 31.5   cancel

A = arcsin [(9/5)sin 31.5]   take the arcsin of each side

A = 70   use calculator

A + B + C = 180   use the triangle sum theorem

70 + B + 31.5 = 180   make substitutions

B + 101.5 = 180   combine like terms

   -101.5   -101.5   subtract 101.5 from each side
_____

B   = 78.5   subtract

results: A = 70; B = 78.5 ; C = 31.5

         a = 9; b = 6 ; c = 5

(6.) a = 3; b = 4; C = 90   here is the problem

    a = 3; b = 4; c = 5   [3,4,5 pythagorean triple]

    tan A = a/b   use this equation to find A

    tan A = 3/4   make substitutions

A = arctan (3/4)   take the arctan of each side

A = 37   use calculator

A + B + C = 180   use the triangle sum theorem

37 + B + 90 = 180   make substitutions

B + 127 = 180   combine like terms

-127   -127   subtract 127 from each side
_____
B   = 53   subtract

results: A = 37; B = 53;   C = 90

a = 3; b = 4; c = 5

(7.) a = 675; A = 48; C = 90

A + B + C = 180 use the triangle sum theorem

48 + B + 90 = 180   make substitutions

B + 138 = 180   combine like terms

- 138 -138   subtract 138 from each side
_____
B   = 42   subtract

tan B = b/a use this equation to find b

tan 42 = b/675   make substitutions

675 tan 42 = b   multiply each side by 675

b = 608   use calculator

sin A = a/c   use this equation to find c

sin 48 = 675/c   make substitutions

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   c sin 48 = 675            multiply each side by c and cancel
_____   _____
   sin 48      sin 48        divide each side by this

  c = (675)/(sin 48)       cancel

  c = 908         use calculator

results:  A = 48;   B = 42;   C = 90

        a = 675;   b = 608;   c = 908



(8.)   a = 137;   c = 78;    C = 23

      sin A       sin C
   _____  =  _____
      a           c          use the law of sines

      sin A       sin 23
   _____  =  _____
      137          78        make substitutions

    78 sin A = 137 sin 23       cross multiply
_____   _____
       78          78        divide each side by 78

   sin A = (137 sin 23)/(78)       cancel

    A = arcsin [(137 sin 23)/(78)]  take arcsin of each side

  A = 43.33            use calculator

  A = 180 - 43.33     subtract from 180

  A = 136.66          subtract

case 1:
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A + B + C = 180   use the triangle sum theorem

43.33 + B + 23 = 180   make substitutions

B + 66.333 = 180   combine like terms

  - 66.333 - 66.333 subtract 66.333 from each side
_____
 B   =   113.66666   subtract

b² = a² + c² - 2ac cos B use the law of cosines to find b

b² = 137² + 78² - 2(137)(78)cos 133.66666 make substitutions

b = 199   use calculator

results: A = 43.33 ;   B = 133.66 ; C = 23

        a = 137;   b = 199; c = 78

case 2:

  A + B + C = 180 use the triangle sum theorem

136.66 + B + 23 = 180   make substitutions

B + 159.66 = 180   combine like terms

 -159.66   -159.66   subtract this from each side
_____
 B   =   20.33   subtract

b² = a² + c² - 2ac cos B   use the law of cosines

b² = (137)² + (78)² - 2(137)(78)cos 20.33 make substitutions

b = 69   use calculator

results: A = 136.66 ;   B = 20.33 ;   C = 23

        a = 137; b = 69 ;   c = 78

(9.)   a = 43;   b = 32;   B = 67   here is the problem

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$ use the law of sines

$$\frac{\sin A}{43} = \frac{\sin 67}{32}$$ make substitutions

32 sin A = 43 sin 67   cross multiply

$$\frac{32}{32} \quad \frac{32}{32}$$ divide each side by 32

sin A = (43/32)sin 67   cancel

A = arcsin [(43/32)sin 67]   take arcsin of each side

[no solution]

(10.)   b = 16;   c = 11;   A = 42   here is the problem

$a^2 = b^2 + c^2 - 2bc \cos A$ use the law of cosines

$a^2 = 16^2 + 11^2 - 2(16)(11) \cos 42$ make substitutions

a = 10.7   use calculator

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$ use the law of sines

$$\frac{\sin 42}{10.7} = \frac{\sin B}{16}$$ make substitutions

10.7 sin B = 16 sin 42   cross multiply

$$\frac{10.7}{10.7} \quad \frac{10.7}{10.7}$$ divide each side by 10.7

sin B =   (16/10.7)sin 42   cancel

B = arcsin [(16/10.7)sin 42   take arcsin of each side

B = 90   use calculator

results: A = 42; B = 90; C = 48

     a = 10.7 ;   b = 16;   c = 11

(11.) b = 47; A = 20; C = 153

      A + B + C = 180   use the triangle sum theorem

     20 + B + 153 = 180   make substitutions

       B + 173 = 180   combine like terms

        -173 -173   subtract 173 from each side
       _____
        B   = 7   subtract

      sin A   sin B
     _____ = _____
        a   b   use the law of sines

       sin 20   sin 7
      _____ = _____
         a   47   make substitutions

      a sin 7 = 47 sin 20   cross multiply
      _____ _____
       sin 7   sin 7   divide each side by sin 7

      a = (47 sin 20)/(sin 7)   cancel

      a = 132   use calculator

      c^2 = a^2 + b^2 - 2ab cos C   use the law of cosines to find c

       c^2 = 132^2 + 47^2 - 2(132)(47)cos 153   make substitutions

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      c = 175        use calculator

results:  A = 20;   B = 7 ;   C = 153

          a = 132;  b = 47;   c = 175
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