

Solve each triangle:

$$(1.) \quad a = 13; \quad A = 41^\circ; \quad B = 75^\circ$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{use the law of sines}$$

$$\frac{\sin 41^\circ}{13} = \frac{\sin 75^\circ}{b} \quad \text{make substitutions}$$

$$b \sin 41^\circ = 13 \sin 75^\circ \quad \text{cross multiply}$$

$$\frac{b \sin 41^\circ}{\sin 41^\circ} = \frac{13 \sin 75^\circ}{\sin 41^\circ} \quad \text{divide each side by } \sin 41^\circ$$

$$b = 19 \quad \text{use calculator and cancel}$$

$$A + B + C = 180^\circ \quad \text{use the triangle sum theorem}$$

$$41^\circ + 75^\circ + C = 180^\circ \quad \text{make substitutions}$$

$$C + 116^\circ = 180^\circ \quad \text{combine like terms}$$

$$-116^\circ \quad -116^\circ \quad \text{subtract 116 from each side}$$

$$\underline{C = 64} \quad \text{subtract}$$

$$\frac{\sin C}{c} = \frac{\sin A}{a} \quad \text{use the law of sines again}$$

$$\frac{\sin 64^\circ}{c} = \frac{\sin 41^\circ}{13} \quad \text{make substitutions}$$

$$c \sin 41^\circ = 13 \sin 64^\circ \quad \text{cross multiply}$$

$$\underline{\sin 41^\circ \quad \sin 41^\circ} \quad \text{divide each side by } \sin 41^\circ$$

$$c = 17.8 \quad \text{use calculator and cancel}$$

results: A = 41; B = 75; C = 64

$$a = 13 ; b = 19 ; c = 17.8$$

(2.) A = 71 ; a = 20 ; C = 62

A + B + C = 180 use the triangle sum theorem

$$71 + B + 62 = 180 \text{ make substitutions}$$

$$B + 133 = 180 \text{ combine like terms}$$

$$-133 -133 \text{ subtract 133 from each side}$$

$$\underline{\underline{B = 47}} \text{ subtract}$$

$\frac{\sin A}{a} = \frac{\sin B}{b}$ use the law of sines

$$\frac{\sin 71}{20} = \frac{\sin 47}{b}$$

make substitutions

$$b \sin 71 = 20 \sin 47 \text{ cross multiply}$$

$$\underline{\underline{\sin 71 \quad \sin 71}} \text{ divide each side by this}$$

$$b = 15.5 \text{ use calculator and cancel}$$

$$\frac{\sin A}{a} = \frac{\sin C}{c} \text{ use the law of sines}$$

$$\frac{\sin 71}{20} = \frac{\sin 62}{c} \text{ make substitutions}$$

$$c \sin 71 = 20 \sin 62 \text{ cross multiply}$$

$$\sin 71 \quad \sin 71 \quad \text{divide each side by } \sin 71$$

$$c = 18.7 \quad \text{use calculator and cancel}$$

$$\text{results: } A = 71; \quad B = 47; \quad C = 62$$

$$a = 20; \quad b = 15.5; \quad c = 18.7$$

$$(3.) \quad A = 71; \quad B = 42; \quad c = 15$$

$$A + B + C = 180 \quad \text{use the triangle sum theorem}$$

$$71 + 42 + C = 180 \quad \text{make substitutions}$$

$$C + 113 = 180 \quad \text{combine like terms}$$

$$-113 \quad -113 \quad \text{subtract 113 from each side}$$

$$\underline{\quad C \quad} = \underline{\quad 67 \quad} \quad \text{subtract}$$

$$\frac{\sin A}{a} = \frac{\sin C}{c} \quad \text{use the law of sines}$$

$$\frac{\sin 71}{a} = \frac{\sin 67}{15} \quad \text{make substitutions}$$

$$a \sin 67 = 15 \sin 71 \quad \text{cross multiply}$$

$$\underline{\quad \sin 67 \quad} \quad \underline{\quad \sin 67 \quad} \quad \text{divide each side by } \sin 67$$

$$a = 15.4 \quad \text{use calculator and cancel}$$

$$\frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{use the law of sines again}$$

$$\frac{\sin 42}{b} = \frac{\sin 67}{15} \quad \text{make substitutions}$$

$$b \sin 67 = 15 \sin 42 \quad \text{cross multiply}$$

$$\frac{\sin 67}{\sin 67} \quad \frac{\sin 67}{\sin 67} \quad \text{divide each side by } \sin 67$$

$$b = 10.9 \quad \text{use calculator and cancel}$$

results: A = 71 ; B = 42 ; C = 67

$$a = 15.4 ; b = 10.9 ; c = 15$$

(4.) a = 12 ; B = 110; C = 35

$$A + B + C = 180 \quad \text{use the triangle sum theorem}$$

$$A + 110 + 35 = 180 \quad \text{make substitutions}$$

$$A + 145 = 180 \quad \text{combine like terms}$$

$$-145 \quad -145 \quad \text{subtract 145 from each side}$$

$$\frac{A}{A} = 35 \quad \text{subtract}$$

c = 12 [c must equal the same as a, because

$\angle A = \angle C$] [opposite angles are = . (geometry)]

[isosceles triangle]

$$b^2 = a^2 + b^2 - 2ab \cos B \quad \text{use the law of cosines}$$

$$b^2 = (12)^2 + (12)^2 - 2(12)(12)\cos 110 \quad \text{make substitutions}$$

$$b^2 = 386.5 \quad \text{use calculator}$$

$$b = 19.66 \quad \text{take square roots}$$

results: A = 35 ; B = 110 ; C = 35

a = 12 ; b = 19.66 ; c = 12

(5.) b = 503 ; A = 15 ; B = 105

$$\frac{\sin B}{b} = \frac{\sin A}{a} \quad \text{use the law of sines}$$

$$\frac{\sin 105}{503} = \frac{\sin 15}{a} \quad \text{make substitutions}$$

$$a \sin 105 = 503 \sin 15 \quad \text{cross multiply}$$

$$\frac{a}{\sin 105} = \frac{503}{\sin 15} \quad \text{divide each side by sin 105}$$

$$a = 134.8 \quad \text{use calculator and cancel}$$

$$A + B + C = 180 \quad \text{use the triangle sum theorem}$$

$$15 + 105 + C = 180 \quad \text{make substitutions}$$

$$C + 120 = 180 \quad \text{combine like terms}$$

$$-120 \quad -120 \quad \text{subtract 120 from each side}$$

$$C = 60 \quad \text{subtract}$$

$$\frac{\sin C}{c} = \frac{\sin B}{b} \quad \text{use the law of sines}$$

$$\frac{\sin 60}{c} = \frac{\sin 105}{503} \quad \text{make substitutions}$$

$$c \sin 105 = 503 \sin 60 \quad \text{cross multiply}$$

$$\frac{c}{\sin 105} = \frac{503}{\sin 60} \quad \text{divide each side by sin 105}$$

$c = 451$ use calculator and cancel

results: $A = 15$; $B = 105$; $C = 60$

$a = 134.8$; $b = 503$; $c = 451$

(6.) $B = 125$; $A = 28$; $b = 14$

$A + B + C = 180$ use the triangle sum theorem

$28 + 125 + C = 180$ make substitutions

$C + 153 = 180$ combine like terms

$-153 -153$ subtract 153 from each side

$$\underline{\underline{C = 27}} \quad \text{subtract}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{use the law of sines}$$

$$\frac{\sin 28}{a} \quad \frac{\sin 125}{14}$$

make substitutions

$$a \sin 125 = 14 \sin 28 \quad \text{cross multiply}$$

$$\frac{\sin 125}{\sin 125} \quad \frac{\sin 125}{\sin 125} \quad \text{divide each side by } \sin 125$$

$$a = 8 \quad \text{use calculator and cancel}$$

$$\frac{\sin C}{c} = \frac{\sin B}{b} \quad \text{use the law of sines}$$

$$\frac{\sin 27}{c} \quad \frac{\sin 125}{14} \quad \text{make substitutions}$$

$$c \sin 125 = 14 \sin 27 \quad \text{cross multiply}$$

$$\frac{\sin 125}{\sin 125} \quad \frac{\sin 125}{\sin 125} \quad \text{divide each side by } \sin 125$$

$$c = 7.76 \quad \text{use calculator and cancel}$$

$$\text{results: } A = 28 ; B = 125 ; C = 27$$

$$a = 8 ; b = 14 ; c = 7.76$$

$$(7.) c = 16.5 ; A = 38 ; C = 54$$

$$A + B + C = 180 \quad \text{use the triangle sum theorem}$$

$$38 + B + 54 = 180 \quad \text{make substitutions}$$

$$B + 92 = 180 \quad \text{combine like terms}$$

$$- 92 \quad -92 \quad \text{subtract 92 from each side}$$

$$\frac{B}{B} = \frac{88}{88} \quad \text{subtract}$$

$$\frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{use the law of sines}$$

$$\frac{\sin 88}{b} = \frac{\sin 54}{16.5} \quad \text{make substitutions}$$

$$b \sin 54 = 16.5 \sin 88 \quad \text{cross multiply}$$

$$\frac{\sin 54}{\sin 54} \quad \frac{\sin 54}{\sin 54} \quad \text{divide each side by } \sin 54$$

$$b = 204 \quad \text{use calculator and cancel}$$

$$\frac{\sin A}{a} = \frac{\sin C}{c} \quad \text{use the law of sines}$$

$$\frac{\sin 38}{a} = \frac{\sin 54}{16.5}$$

make substitution

$$a \sin 54 = 16.5 \sin 38$$

cross multiply

$$\frac{\sin 54}{\sin 54} = \frac{16.5}{\sin 54}$$

divide each side by $\sin 54$

$$a = 12.5$$

use calculator and cancel

$$\text{results: } A = 38 ; B = 88 ; C = 54$$

$$a = 12.5 ; b = 204 ; c = 16.5$$

$$(8.) b = 14.4 ; A = 72 ; C = 19$$

$$A + B + C = 180$$

use the triangle sum theorem

$$72 + B + 19 = 180$$

make substitutions

$$B + 91 = 180$$

combine like terms

$$-91 -91$$

subtract 91 from each side

$$\frac{B}{B} = \frac{89}{89}$$

subtract

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

use the law of sines

$$\frac{\sin 72}{a} = \frac{\sin 89}{14.4}$$

make substitutions

$$a \sin 89 = 14.4 \sin 72$$

cross multiply

$$\frac{\sin 89}{\sin 89} = \frac{14.4}{\sin 89}$$

divide each side by $\sin 89$

$$a = 13.7$$

use calculator and cancel

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

use the law of sines

$$\frac{\sin 19}{c} = \frac{\sin 89}{14.4}$$

make substitutions

$$c \sin 89 = 14.4 \sin 19$$

cross multiply

$$\frac{\sin 89}{\sin 89} = \frac{14.4}{\sin 89}$$

divide each side by $\sin 89$

$$c = 4.7$$

use calculator and cancel

$$\text{results: } A = 72; B = 89; C = 19$$

$$a = 13.7; b = 14.4; c = 4.7$$

$$(9.) b = 224; A = 21; B = 84$$

$$A + B + C = 180$$

use the triangle sum theorem

$$21 + 84 + C = 180$$

make substitutions

$$C + 105 = 180$$

combine like terms

$$-105 -105$$

subtract 105 from each side

$$\frac{C}{C} = 75$$

subtract

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

use the law of sines

$$\frac{\sin 75}{c} = \frac{\sin 84}{224}$$

make substitutions

$$c \sin 84 = 224 \sin 75$$

cross multiply

$$\sin 84 \quad \sin 84 \quad \text{divide each side by } \sin 84$$

$$c = 217.5 \quad \text{use calculator}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{use the law of sines}$$

$$\frac{\sin 21}{a} = \frac{\sin 84}{224} \quad \text{make substitutions}$$

$$a \sin 84 = 224 \sin 21 \quad \text{cross multiply}$$

$$\frac{\sin 84}{\sin 84} \quad \frac{\sin 84}{\sin 84} \quad \text{divide each side by } \sin 84$$

$$a = 80.7 \quad \text{use calculator and cancel}$$

results: $A = 21$; $B = 84$; $C = 75$

$$a = 80.7 ; b = 224 ; c = 217.5$$

$$(10.) \quad c = 916 ; A = 15 ; B = 60$$

$$A + B + C = 180 \quad \text{use the triangle sum theorem}$$

$$15 + 60 + C = 180 \quad \text{make substitutions}$$

$$C + 75 = 180 \quad \text{combine like terms}$$

$$- 75 - 75 \quad \text{subtract 75 from each side}$$

$$\underline{C = 105} \quad \text{subtract}$$

$$\frac{\sin A}{a} = \frac{\sin C}{c} \quad \text{use the law of sines}$$

$$\frac{\sin 15}{a} = \frac{\sin 105}{916} \quad \text{make substitutions}$$

$$a \sin 105 = 916 \sin 15 \quad \text{cross multiply}$$

$$\frac{\sin 105}{\sin 105} = \frac{\sin 15}{\sin 15} \quad \text{divide each side by } \sin 105$$

$$a = 245.5 \quad \text{use calculator and cancel}$$

$$\frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{use the law of sines}$$

$$\frac{\sin 60}{b} = \frac{\sin 105}{916} \quad \text{make substitutions}$$

$$b \sin 105 = 916 \sin 60 \quad \text{cross multiply}$$

$$\frac{\sin 105}{\sin 105} = \frac{\sin 60}{\sin 60} \quad \text{divide each side by } \sin 105$$

$$b = 821 \quad \text{use calculator and cancel}$$

results: A = 15; B = 60 ; C = 105

$$a = 245.5 ; b = 821 ; c = 916$$

$$(11.) \quad A = 101 ; C = 37 ; a = 23$$

$$A + B + C = 180 \quad \text{use the triangle sum theorem}$$

$$101 + B + 37 = 180 \quad \text{make substitutions}$$

$$B + 138 = 180 \quad \text{combine like terms}$$

$$- 138 - 138 \quad \text{subtract 138 from each side}$$

$$\frac{B}{B} = 42 \quad \text{subtract}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{use the law of sines}$$

$$\frac{\sin 101}{23} = \frac{\sin 42}{b}$$

make substitutions

$$b \sin 101 = 23 \sin 42$$

cross multiply

$$\frac{\sin 101}{\sin 101} = \frac{23}{\sin 101}$$

divide each side by sin 101

$$b = 15.7$$

use calculator and cancel

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

use the law of sines

$$\frac{\sin 37}{c} = \frac{\sin 101}{23}$$

make substitutions

$$c \sin 101 = 23 \sin 37$$

cross multiply

$$\frac{\sin 101}{\sin 101} = \frac{23}{\sin 101}$$

divide each side by this

$$c = 14$$

use calculator and cancel

results: $A = 101$; $B = 42$; $C = 37$

$$a = 23 ; b = 15.7 ; c = 14$$

$$(12.) B = 152; b = 95; C = 12$$

$$A + B + C = 180$$

use the triangle sum theorem

$$A + 152 + 12 = 180$$

make substitutions

$$A + 164 = 180$$

combine like terms

$$-164 -164$$

subtract 164 from each side

$$\frac{A}{16} = \frac{16}{16}$$

subtract

$$\sin B \quad \sin A$$

$$\frac{b}{\sin 152} = \frac{a}{\sin 16}$$
 use the law of sines

$$\frac{\sin 152}{95} = \frac{\sin 16}{a}$$
 make substitutions

$$a \sin 152 = 95 \sin 16$$
 cross multiply

$$\frac{\sin 152}{\sin 152} = \frac{\sin 16}{\sin 16}$$
 divide each side by $\sin 152$

$$a = 55.8$$
 use calculator and cancel

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$
 use the law of sines

$$\frac{\sin 152}{95} = \frac{\sin 12}{c}$$
 make substitutions

$$c \sin 152 = 95 \sin 12$$
 cross multiply

$$\frac{\sin 152}{\sin 152} = \frac{\sin 12}{\sin 12}$$
 divide each side by $\sin 152$

$$c = 42$$
 use calculator

results: $A = 16$; $B = 152$; $C = 12$

$$a = 55.8 ; b = 95 ; c = 42$$

$$(13.) a = 150; B = 32; C = 54$$

$$A + B + C = 180$$
 use the triangle sum theorem

$$A + 32 + 54 = 180$$
 make substitutions

$$A + 86 = 180$$
 combine like terms

$$- 86 - 86$$
 subtract 86 from each side

$$A = 94 \quad \text{subtract}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{use the law of sines}$$

$$\frac{\sin 94}{150} = \frac{\sin 32}{b} \quad \text{make substitutions}$$

$$b \sin 94 = 150 \sin 32 \quad \text{cross multiply}$$

$$\frac{\sin 94}{\sin 94} = \frac{\sin 94}{\sin 94} \quad \text{divide each side by } \sin 94$$

$$b = 80 \quad \text{use calculator and cancel}$$

$$\frac{\sin A}{a} = \frac{\sin C}{c} \quad \text{use the law of sines}$$

$$\frac{\sin 94}{150} = \frac{\sin 54}{c} \quad \text{make substitutions}$$

$$c \sin 94 = 150 \sin 54 \quad \text{cross multiply}$$

$$\frac{\sin 94}{\sin 94} = \frac{\sin 94}{\sin 94} \quad \text{divide each side by } \sin 94$$

$$c = 121.6 \quad \text{use calculator and cancel}$$

results: $A = 94$; $B = 32$; $C = 54$

$$a = 150 ; b = 80 ; c = 121.6$$

$$(14.) \quad a = 75 ; A = 18 ; C = 32$$

$$A + B + C = 180 \quad \text{use the triangle sum theorem}$$

$$18 + B + 32 = 180 \quad \text{make substitutions}$$

$$B + 50 = 180 \quad \text{combine like terms}$$

$$- 50 \quad -50 \quad \text{subtract 50 from each side}$$

$$\underline{\underline{B}} \quad = \quad 130 \quad \text{subtract}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{use the law of sines}$$

$$\frac{\sin 18}{75} = \frac{\sin 130}{b} \quad \text{make substitutions}$$

$$b \sin 18 = 75 \sin 130 \quad \text{cross multiply}$$

$$\frac{\sin 18}{\sin 18} \quad \frac{\sin 18}{\sin 18} \quad \text{divide each side by } \sin 18$$

$$b = 186 \quad \text{use calculator and cancel}$$

$$\frac{\sin C}{c} = \frac{\sin A}{a} \quad \text{use the law of sines}$$

$$\frac{\sin 32}{c} = \frac{\sin 18}{75} \quad \text{make substitutions}$$

$$c \sin 18 = 75 \sin 32 \quad \text{cross multiply}$$

$$\frac{\sin 18}{\sin 18} \quad \frac{\sin 18}{\sin 18} \quad \text{divide each side by } \sin 18$$

$$c = 128.6 \quad \text{use calculator and cancel}$$

Solve each triangle:

$$(5.) \quad b = 40 ; \quad c = 45 ; \quad A = 51$$

$$a^2 = b^2 + c^2 - 2bc \cos A \quad \text{use the law of cosines}$$

$$a^2 = (40)^2 + (45)^2 - 2(40)(45)\cos 51 \quad \text{make substitutions}$$

$$a^2 = 1359.44659222 \quad \text{use calculator}$$

$$a = 36.87 \quad \text{take the square root of each side}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{use the law of sines}$$

$$\frac{\sin 51}{36.87} = \frac{\sin B}{40} \quad \text{make substitutions}$$

$$40 \sin 51 = 36.87 \sin B \quad \text{cross multiply}$$

$$\frac{36.87}{36.87} \quad \frac{36.87}{36.87} \quad \text{divide each side by this}$$

$$0.84312 = \sin B \quad \text{use calculator and cancel}$$

$$B = 57.5 \quad \text{take the arcsin of each side}$$

$$A + B + C = 180 \quad \text{use the triangle sum theorem}$$

$$51 + 57.5 + C = 180 \quad \text{make substitutions}$$

$$C + 108.5 = 180 \quad \text{combine like terms}$$

$$- 108.5 \quad -108.5 \quad \text{subtract 108.5 fr ea side}$$

$$\underline{C} \quad = \quad 71.5 \quad \text{subtract}$$

results: $A = 51$; $B = 57.5$; $C = 71.5$

$$a = 36.87 ; b = 40 ; c = 45$$

(16.) $a = 20$; $c = 24$; $B = 47$

$$b^2 = a^2 + c^2 - 2ac \cos B \quad \text{use the law of cosines}$$

$$b^2 = (20)^2 + (24)^2 - 2(20)(24) \cos 47 \quad \text{make substitutions}$$

$$b^2 = 321.28 \quad \text{use calculator}$$

$$b = 18 \quad \text{take the square root of each side}$$

$$\frac{\sin B}{b} = \frac{\sin A}{a} \quad \text{use the law of sines}$$

$$\frac{\sin 47}{18} = \frac{\sin A}{20} \quad \text{make substitutions}$$

$$18 \sin A = 20 \sin 47 \quad \text{cross multiply}$$

$$\frac{18}{18} \frac{\sin A}{18} \quad \text{divide each side by 18}$$

$$\sin A = 0.812615224 \quad \text{use calculator}$$

$$A = 54 \quad \text{take the arcsin of each side}$$

$$A + B + C = 180 \quad \text{use the triangle sum theorem}$$

$$54 + 47 + C = 180 \quad \text{make substitutions}$$

$$C + 101 = 180 \quad \text{combine like terms}$$

$$- 101 - 101 \quad \text{subtract 101 from each side}$$

$$\underline{C = 79} \quad \text{subtract}$$

$$\text{results: } A = 54 ; B = 47 ; C = 79$$

$$a = 20 ; b = 18 ; c = 24$$

$$(17.) \quad a = 5 ; b = 6 ; c = 7 \quad \text{here is the problem}$$

$$c^2 = a^2 + b^2 - 2ab \cos C \quad \text{use the law of cosines}$$

$$(7)^2 = (5)^2 + (6)^2 - 2(5)(6)\cos C \quad \text{make substitutions}$$

$$49 = 25 + 36 - 60\cos C \quad \text{multiply}$$

$$49 = 61 - 60 \cos C \quad \text{combine like terms}$$

$$-49 = -61 + 60 \cos C \quad \text{multiply thru by } -1$$

$$+ 61 \quad +61 \quad \text{add 61 to each side}$$

$$12 = 60 \cos C \quad \text{add}$$

$$60 \cos C = 12 \quad \text{just rearrange like this}$$

$$60 \quad 60 \quad \text{divide each side by 60}$$

$$\cos C = 0.2 \quad \text{divide and cancel}$$

$$C = 78.5 \quad \text{take the arccos of each side}$$

$$\frac{\sin C}{c} = \frac{\sin B}{b} \quad \text{use the law of sines}$$

$$\frac{\sin 78.5}{7} = \frac{\sin B}{6} \quad \text{make substitutions}$$

$$7 \sin B = 6 \sin 78.5 \quad \text{cross multiply}$$

$$7 \quad 7 \quad \text{divide each side by 7}$$

$$\sin B = 0.84 \quad \text{use calculator and cancel}$$

$$B = 57 \quad \text{take the arcsin of each side}$$

$$A + B + C = 180 \quad \text{use the triangle sum theorem}$$

$$A + 57 + 78.5 = 180 \quad \text{make substitutions}$$

$$A + 135.5 = 180 \quad \text{combine like terms}$$

$$-135.5 \quad -135.5 \quad \text{subtract 135.5 from each side}$$

$$\overline{A} = 44.5 \quad \text{subtract}$$

results: $A = 44.5$; $B = 57$; $C = 78.5$

$$a = 5; b = 6; c = 7$$

(18.) $a = 5$; $b = 12$; $c = 13$ here is the problem

$C = 90$ [this is a right triangle]

$\sin A = 5/13$ use this equation to find A

$A = \arcsin(5/13)$ take the arcsin of each side

$A = 22.6$ use calculator

$A + B + C = 180$ use the triangle sum theorem

$22.6 + B + 90 = 180$ make substitutions

$B + 112.6 = 180$ combine like terms

$- 112.6 - 112.6$ subtract 112.6 from each side

$$\overline{B} = 67.4 \quad \text{subtract}$$

results: $A = 22.6$; $B = 67.4$; $C = 90$

$$a = 5; b = 12; c = 13$$

(19.) $b = 16$; $c = 19$; $A = 35$ here is the problem

$a^2 = b^2 + c^2 - 2bc \cos A$ use the law of cosines

$a^2 = (16)^2 + (19)^2 - 2(16)(19)\cos 35$ make substitutions

$a^2 = 119$ use calculator

$a = 11$ take square root of each side

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

use the law of sines

$$\frac{\sin 35}{11} = \frac{\sin B}{16}$$

make substitutions

$$11 \sin B = 16 \sin 35$$

cross multiply

$$\frac{11}{11} \frac{\sin B}{\sin 35} = \frac{16}{11}$$

divide each side by 11

$$\sin B = (16 \sin 35)/11$$

cancel

$$B = \arcsin [(16 \sin 35)/11]$$

take arcsin of each side

$$B = 56.5$$

use calculator

$$A + B + C = 180$$

use the triangle sum theorem

$$35 + 56.5 + C = 180$$

make substitutions

$$C + 91.5 = 180$$

combine like terms

$$- 91.5 - 91.5$$

subtract 91.5 from each side

$$C = 88.5$$

subtract

$$\text{results: } A = 35 ; B = 56.5 ; C = 88.5$$

$$a = 11; b = 16; c = 19$$

$$(20.) b = 13; a = 11; C = 76$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

use the law of cosines

$$c^2 = (11)^2 + (13)^2 - 2(11)(13)\cos 76$$

make substitutions

$$c^2 = 220.8$$

use calculator

$$c = 14.8 \quad \text{take square roots}$$

$$\frac{\sin C}{c} = \frac{\sin A}{a} \quad \text{use the law of sines}$$

$$\frac{\sin 76}{14.8} = \frac{\sin A}{11} \quad \text{make substitutions}$$

$$\frac{14.8 \sin A}{14.8} = \frac{11 \sin 76}{14.8} \quad \text{cross multiply}$$

$$\sin A = (11 \sin 76) / 14.8 \quad \text{cancel}$$

$$A = \arcsin [(11 \sin 76) / 14.8] \quad \text{take arcsin of each side}$$

$$A = 46 \quad \text{use calculator}$$

$$A + B + C = 180 \quad \text{use the triangle sum theorem}$$

$$46 + B + 76 = 180 \quad \text{make substitutions}$$

$$B + 122 = 180 \quad \text{combine like terms}$$

$$\frac{-122 -122}{B} = 58 \quad \text{subtract 122 from each side}$$

results: $A = 46$; $B = 58$; $C = 76$

$$a = 11; b = 13; c = 14.8$$