Prove each identity:

(1.) $\tan x = \sin x \sec x$ here is the problem

$\sin x \sec x$ start on the left side

$= (\sin x) * (1/\cos x)$ reciprocal identity

$= (\sin x)/(\cos x)$ multiply

$= \tan x$ definition of tangent

(2.) $\cot x = \cos x \csc x$ here is the problem

$\cos x \csc x$ start on the right side

$= (\cos x)/1 * (1/\sin x)$ reciprocal identity

$= \cos x/\sin x$ multiply

$= \cot x$ definition of cotangent

(3.) $\tan^2 x = (1 - \cos^2 x)/(\cos^2 x)$ here is the problem

$(1 - \cos^2 x)/(\cos^2 x)$ start on the left side

$= (\sin^2 x)/(\cos^2 x)$ pythagorean identity

$= \tan^2 x$ definition of tan

(4.) $\sec^2 x = (\sin^2 x + \cos^2 x)/(\cos^2 x)$ here is the problem

$(\sin^2 x + \cos^2 x)/(\cos^2 x)$ start on the right side

$= 1/(\cos^2 x)$ pythagorean identity

$= \sec^2 x$ reciprocal identity

(5.) $\tan^2 x = \sec^2 x - 1$ here is the problem

$\sec^2 x - 1$ start on the right side

= $[1/\cos^2 x] - 1$ reciprocal identity

= $[1/\cos^2 x) - (\cos^2 x/\cos^2 x)$ change to this form of 1

= $(1 - \cos^2 x)/(\cos^2 x)$ subtract fractions

= $(\sin^2 x)/(\cos^2 x)$ pythagorean identity

= $\tan^2 x$ definition of tangent

(6.) $\cot^2 x = \csc^2 x - 1$ here is the problem

$\csc^2 x - 1$ start on the right side

= $[1/\sin^2 x) - (\sin^2 x/\sin^2 x)$ this form of 1

= $(1 - \sin^2 x)/(\sin^2 x)$ subtract fractions

= $(\cos^2 x)/(\sin^2 x)$ pythagorean identity

= $\cot^2 x$ definition of cot

(7.) $\csc x = (\cot x)/(\cos x)$ here is the problem

$(\cot x)/(\cos x)$ start on the right side

= $(\cos x)/(\sin x) * 1/(\cos x)$ definition of cot

and multiply by the reciprocal

= $1/\sin x$ cancel and multiply

= $\csc x$ reciprocal identity

(8.) $[1/(\sec^2 x)] + [1/(\csc^2 x)] = 1$ here is the problem

$[1/\sec^2 x] + [1/\csc^2 x)$ start on the left side

= $\cos^2 x + \sin^2 x$ reciprocal identities

$= 1$ pythagorean identity

(9.) $\csc^2 x \tan^2 x - 1 = \tan^2 x$ here is the problem

$\csc^2 x \tan^2 x - 1$ start on the left side

$= (\tan^2 x)(\csc^2 x - \cot^2 x)$ factor

$= (\tan^2 x)(1 - \cos^2 x)/(\sin^2 x)$ factor

$= (\tan^2 x)(\sin^2 x)/(\sin^2 x)$ pythagorean identity

$= \tan^2 x$ cancel

(10.) $(\sec x)/(\cos x) - (\tan x)/(\cot x) = 1$

$(\sec x)/(\cos x) - (\tan x)/(\cot x)$ start on the left

$= \sec^2 x - \tan^2 x$ multiply by reciprocals of the bottom

$= (1 - \sin^2 x)/(\cos^2 x)$ factor like this

$= (\cos^2 x)/(\cos^2 x)$ pythagorean identity

$= 1$ cancel

(11.) $(1 - \tan x)^2 = \sec^2 x - 2 \tan x$ here is the problem

$(1 - \tan x)^2$ start on the left side

$= 1 - 2 \tan x + \tan^2 x$ square the binomial

$= 1 + \tan^2 x - 2 \tan x$ rearrange terms

$= \sec^2 x - 2 \tan x$ pythagorean identity

(12.) $(1 - \sin^2 x)(1 + \tan^2 x) = 1$ here is the problem

$= (\cos^2 x)(\sec^2 x)$ pythagorean identity

$= 1$ multiply reciprocals

(13.) $(\cos^2 x)/(\sin x) + \sin x = \csc x$ here is the problem

$(\cos^2 x)/(\sin x) + \sin x$ start on the left side

$= (\cos^2 x + \sin^2 x)/(\sin x)$ factor like this

$= 1/\sin x$ pythagorean identity

$= \csc x$

(14.) $\tan x + \cot x = \sec x \csc x$ here is the problem

$\tan x + \cot x$ start on the left side

$= \dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x}$ add fractions over this common denominator

$= 1/(\sin x \cos x)$ pythagorean identity

$= \csc x \sec x$ reciprocal identities

$= \sec x \csc x$ just rearrange

(15.) $(\tan x)/(1 - \cos^2 x) = (\sec x)/(\sin x)$ here is the problem

$(\tan x)/(1 - \cos^2 x)$ start on the left side

$= (\tan x)/(\sin^2 x)$ pythagorean identity

$= \dfrac{\sin x}{\cos x} * \dfrac{1}{\sin^2 x}$ write as fractions

$$= \frac{1}{\cos x} * \frac{1}{\sin x} \quad \text{cancel}$$

$= (\sec x)/(\sin x)$ reciprocal identity

(16.) $(\cot x)/(\cos x) + (\sec x)/(\cot x) = \sec^2 x \csc x$

$(\cot x)/(\cos x) + (\sec x)/(\cot x)$ start on the left side

$= (\cot x)(\sec x) + (\sec x)(\tan x)$ multiply by reciprocals

$= 1/(\sin x) + (\sin x)/(\cos^2 x)$

$$= \frac{(\sin x)}{\sin^2 x} + \frac{\sin x}{\cos^2 x}$$ multiply by sin x, top and bottom

$$= \frac{(\sin x)(\cos^2 x) + \sin^3 x}{\sin^2 x \cos^2 x}$$ add fractions over this common denominator

$$= \frac{(\sin x)(\cos^2 x + \sin^2 x)}{\sin^2 x \cos^2 x}$$ factor on top

$$= \frac{(\sin x)(1)}{\sin^2 x \cos^2 x}$$ pythagorean identity

$= \sec^2 x \csc x$ reciprocal identities

(17.) $(\cos x - \sin x)/(\cos x) = 1 - \tan x$ here is the problem

$$\frac{(\cos x - \sin x)}{\cos x}$$ start on the left side

= 1 - tan x   divide thru by cos x

(18.) (cot x + 1)/(cot x) = 1 + tan x

(cot x + 1)/(cot x)   start on the left side

= 1 + tan x   divide thru by cot x

(19.) (tan x)(tan x + cot x) = $\sec^2 x$ here is the problem

(tan x)(tan x + cot x)   start on the left side

= $\tan^2 x + 1$ multiply thru parentheses

= $\sec^2 x$ pythagorean identity

(20.) (sec x - tan x)(sec x + tan x) = 1 here is the problem

= $\sec^2 x - \tan^2 x$ foil multiply combine like terms

= 1   pythagorean identity