(1.)   A woman invested a total of $40,000 in three business

ventures: a bowling alley, a diner, and a laundrymat. In

a recent year, the bowling alley returned a profit of 3%,

the diner returned a profit of 8%, and the laundrymat

returned a profit of 12% . Total income from the 3

investments was 2850. The income from the diner was

the same as the income from the laundrymat. Find the

amount invested in each venture.

b + d + L = 40,000

.08d = .12L

$\frac{}{.08}$ $\frac{}{.08}$ divide each side by .08

d = 1.5L   divide and cancel

.03b + .08d + .12L = 2850   here is the problem

-0.03b - 0.03d - .03L = -1200 multiply thru by -0.03
_____
        0.05d + 0.09L = 1650   subtract equations

      0.05(1.5L) + 0.09L = 1650   replace d with 1.5L

0.075L + 0.09L = 1650   multiply

0.165L = 1650   combine like terms

$\frac{}{0.165}$ $\frac{}{0.165}$ divide each side by this

L = 10,000   divide and cancel

d = 1.5(10,000)   replace L with 10,000

d = 15,000   multiply

```
    b + 15,000 + 10,000 = 40,000    make substitutions

   b + 25,000 = 40,000              combine like terms

     -25,000  -25,000    subtract 25,000 fr ea side
   _____
     b    =   15,000          subtract
```

results: b = 15,000 ; d = 15,000 ;   L = 10,000

(2.)   A ceramics artisan makes cups, pitchers, and vases. The

    cost is $0.60 per cup, $0.25 per pitcher, and $2.00 per

    vase. His total cost for the 15 items made per day is

    $9.00 . It takes the artisan 20 minutes to make a cup, 10

    minutes to make a pitcher, and 30 minutes to make a vase.

    The total time worked per day is 4 hours. How many items

    of each type are produced each day?

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    .60c + 0.25p + 2v = 9

    20c + 10p + 30v = 240

     c + p + v = 15                        here is the problem

    10c + 10p + 10v = 150   multiply that equation thru by 10

    20c + 10p + 30v = 240   put this here
   _____
    10c      +  20v = 90     subtract upsidown

     c + 2v = 9              divide thru by 10, cancel

    0.25c + 0.25p + 0.25v = 3.75  multiply eq 3 thru by 0.25

    .60c + 0.25p +  2v  =   9   put this here
   _____
     .35c          +  1.75v = 5.25    subtract upsidown
```

```
  -  .35c          -  0.70v = -3.15    multiply c+2v=9thru by 0.35
_____

                  1.05v =  2.10          subtract
                  _____ _____
                   1.05    1.05      divide ea side by 1.05

                   v = 2             divide and cancel

   c + 2(2) = 9    replace v with 2

   c + 4 = 9            multiply

       -4   -4            subtract 4 from each side
_____
         c = 5             subtract

     5 + p + 2 = 15    replace c and v with 5 and 2

       p + 7 = 15      combine like terms

       -   7  -7       subtract 7 from each side
_____
         p =    8    subtract
```

results: c = 5; p = 8; v = 2

(3.)   The sum of the digits of a certain three-digit number is

       14. The sum of the hundreds digit and the tens digit is

       11. When the digits are reversed, the new number is 198

       less than the original number. Find the original number.

        $h + t + u = 14$

        $h + t = 11$

       $100u + 10t + h = 100h + 10t + u - 198$

```
        -10t              - 10t   subtract 10t fr ea side
_____
  100u         + h = 100h      +   u - 198          subtract

   -u                                 -u      subtract u fr ea side
_____
   99u         + h  = 100h      - 198           subtract

           - h    - h          subtract h from each side
_____
  99u                =  99h - 198          subtract
____                   ___  ____
 99                     99    99          divide thru by 99

  u  = h - 2                  divide and cancel

   h + t + u = 14      use this equation

  h + (11 - h) + h - 2 = 14   make substitutions

       h + 9 = 14                 combine like terms

         - 9  - 9                 subtract 9 from each side
  _____
         h  =  5                  subtract

    u = 5 - 2      replace h with 5

   u = 3              subtract

   t = 11 - 5         replace h with 5

   t = 6              subtract
result:  the number was 563
```

(5.) The sum of the 3 digits of a number is 13.   If the

    number, decreased by 8, is divided by the sum of the

   units digit and tens digit, the quotient is 25.   When

the digits are reversed, the new number exceeds the original

number by 99. Find the original number.

h + t + u = 13

$$\frac{100h + 10t + u - 8}{t + u} = 25$$

100u + 10t + h = 100h + 10t + u + 99

[here is the problem]

100h + 10t + u - 8 = 25t + 25u   multiply ea side by t + u

        -25t   -25t   subtract 25t fr ea side
_____
 100h - 15t + u - 8 =   25u   subtract

          -25u   -25u   subtract 25u fr ea side
_____
 100h - 15t - 24u - 8 = 0   subtract

99u = 99h + 99

u = h + 1   divide thru by 99

h + t + u = 13

100h - 15t - 24u = 8

u = h + 1   here is the problem

100h - 15t - 24u = 8   put this here

15h + 15t + 15u = 195 multiply h+t+u=13 thru by 15
_____
115h   - 9u = 203   subtract equations

115h - 9(h + 1) = 203   replace u with h + 1

115h - 9h - 9 = 203   multiply thru parentheses

106h - 9 = 203   combine like terms

    + 9   + 9   add 9 to each side
_____
 106h   = 212   add

____   ____
 106   106   divide each side by 106

      h = 2   divide and cancel

   u = 2 + 1   replace h with 2

  u = 3   add

  2 + t + 3 = 13   replace u and h with 3 and 2

   t + 5 = 13   combine like terms

     -5   -5   subtract 5 from each side
_____
      t = 8   subtract

result: the number is 283

(6.) Find the 3 angles of a triangle if the sum of the 1st and

     twice the 2nd equals the 3rd angle, and if 4 times the

    2nd angle is 15° more than the 3rd.

    A + B + C = 180

    A + 2B = C

     4B = C + 15   here is the problem

   A + 2B - C = 0   put this here

   A + B + C = 180   put this here
_____
    2A + 3B = 180   add equations

```
A + 2B = 4B - 15

   -2B   -2B                  subtract 2B from each side
_____
  A        = 2B - 15         subtract

  2(2B - 15) + 3B = 180    replace A with 2B - 15

  4B - 30 + 3B = 180         multiply thru parentheses

   7B - 30 = 180          combine like terms

      +   30  +30         add 30 to each side
_____
     7B          = 210        add

     ___        _____
      7           7           divide each side by 7

       B = 30               divide and cancel

  A = 2(30) - 15    replace B with 30

  A = 45               multiply and subtract

  45 + 30 + C = 180  replace A and B with 45 and 30

   75 + C = 180            combine like terms

    - 75      -75        subtract 75 from each side
_____
        C = 105           subtract
```

results: A = 45; B = 30; C = 105

(7.) The perimeter of a triangle is 38 cm. The longest side

   is 4 cm less than twice the middle side. It is also

   2 cm less than the sum of the other 2 sides. Find the

3 sides.

a + b + c = 38

c = 2b - 4

c = a + b - 2

+2   + 2   add 2 to each side
_____
c + 2 = a + b   add

-c   - c subtract c from each side
_____
    2 = a + b - c   subtract

 a + b - c = 2   just rearrange like this

 a + b + c = 38   put this here
_____
 2a + 2b = 40   add equations
___ ____   ___
 2   2   2   divide thru by 2

a + b = 20   divide and cancel

c + 2 = 20   replace a + b with c + 2

   -2   -2   subtract 2 from each side
_____
   c   = 18   subtract

   c = 2b - 4   use this equation to find b

 18 = 2b - 4   replace c with 18

 + 4   +   4 add 4 to each side
_____
  22 = 2b   add

 __   ___
 2   2   divide each side by 2

 11 = b   divide and cancel

```
a + b + c = 38     use this equation to find a

a + 11 + 18 = 38     replace b and c with 11 and 18

 a + 29 = 38         combine like terms

    -29    -29         subtract 29 from each side
_____
   a     =   9          subtract

results:  a = 9;   b = 11;   c = 18


(8.)   3 trucks together haul 78 cu. m, 81 cu. m, and 69 cu. m

       in 3 days.  Find the capacity of each truck if they haul

       the following number of loads each day.  1st day:

       4;3;and 5 loads.  2nd day:  5;4; and 4 loads.  3rd

       day:  3,5  and 3 loads.

       4a + 3b + 5c = 78

       5a + 4b + 4c = 81

       3a + 5b + 3c = 69                here is the problem

       15a + 12b + 12c = 243   multiply eq 2 thru by 3

       15a + 25b + 15c = 345    multiply eq 3 thru by 5
       _____
             13b + 3c = 102    subtract equations

       20a + 15b + 25c = 390   multiply eq 1 thru by 5

       20a + 16b + 16c = 324   multiply eq 2 thru by 4
       _____
             -b + 9c =   66     subtract equations

           -13b + 117c = 858    multiply thru by 13
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```
          13b + 3c = 102    put this here
  _____
               120c = 960      add equations

               ____  _____
               120   120     divide each side by 120

                c = 8            divide and cancel

    -b + 9(8) = 66           replace c with 8

  -b + 78 = 66                    multiply

  b - 78 = -66           multiply thru by -1

   +  78  +78   add 78 to each side
  _____
   b      = 12          add

  4a + 3b + 5c = 78   use this equation to find a

 4a + 3(12) + 5(8) = 78    replace b and c with 12 & 8

 4a + 36 + 40 = 78          multiply

   4a + 76 = 78      combine like terms

        -76  -76          subtract 76 from each side
   _____
    4a     =    2          subtract

   ____        ____
    4          4          divide each side by 4

   a = 1/2         reduce and cancel
```

results: a = 1/2 ; b = 12 ; c = 8

(9.) Mary and Tiffany can do a job in 10 days. Mary and

     Cecile in 12 days, and Tiffany and Cecile in 20 days.

     How long would it take each person working alone?

$$\frac{10}{m} + \frac{10}{t} = 1$$

$$\frac{12}{m} + \frac{12}{c} = 1$$

$$\frac{20}{t} + \frac{20}{c} = 1 \qquad\qquad \text{here is the problem}$$

(10/t) = 1 - (10/m)   subtract 10/m from each side

(10/t) = (m - 10)/m   subtract fractions

t/10 = m/(m - 10)   take reciprocals

t = (10m)/(m - 10)   multiply each side by 10, cancel

(20/t) = 1 - (20/c)   subtract from each side

(20/t) = (c - 20)/c   subtract fractions

t/20 = c/(c - 20)   take reciprocals

t = (20c)/(c - 20)   multiply each side by 20, cancel

(20c)/(c - 20) = (10m)/(m - 10)   set equal to each other

20cm - 200c = 10cm - 200m   cross multiply

$$\frac{}{10} \quad \frac{}{10} \quad \frac{}{10} \quad \frac{}{10}$$ divide thru by 10

2cm - 20c = cm - 20m   divide and cancel

cm = 20c - 20m   add 20c to each side, subt cm fr ea side

$$\frac{12}{m} + \frac{12}{c} = 1$$

12c + 12m = cm   multiply thru by cm and cancel

12c + 12m = 20c - 20m   set equations equal to each other

    +12m   + 12m   add 12m to each side
_____
12c   = 20c - 8m   add

-20c   -20c   subtract 20c from each side
_____
 -8c   =   -8m   subtract

 ____   ____
  -8   -8   divide each side by -8

   c = m   cancel

  12c + 12(c) = c(c)   replace m with c

   24c = c^2 multiply

  -24c   - 24c   subtract 24c from each side
_____
   0 = c^2 - 24c   subtract

   0 = c(c - 24)   factor

 c - 24 = 0   set this factor equal to 0

  + 24 +24   add 24 to each side
_____
    c = 24

    m = 24   because c = m

  t = (20c)/(c - 20)   use this equation to find t

  t = (20*24)/(24 - 20)   replace c with 24

  t =(480)/(4)   multiply and subtract

 t = 120   divide

results: c = 24 ; m = 24; t = 120

(10.) John, Joe, and Art working together can complete a job

       in 3 days. John and Joe woking together take 3.75 days

         to complete the job.   Joe and Art working together

         take 6 days to complete the job.   How many days would

         it take each person working alone to complete the job?

$$\frac{3}{N} + \frac{3}{J} + \frac{3}{A} = 1$$

$$\frac{3.75}{N} + \frac{3.75}{J} = 1$$

$$\frac{6}{J} + \frac{6}{A} = 1 \qquad\qquad\qquad here\ is\ the\ problem$$

$(6/J) = 1 - (6/A)$ subtract from each side

$(6/J) = (A - 6)/A$ subtract fractions

$J/6 = A/(A - 6)$ take reciprocals

$J = (6A)/(A - 6)$ multiply each side by 6, cancel

$$\frac{3.75}{N} = 1 - (3.75/J)\quad subtract\ from\ each\ side$$

$$\frac{3.75}{N} = (J - 3.75)/J \qquad subtract\ fractions$$

$N/3.75 = J/(J - 3.75)$ take reciprocals

N = (3.75J)/(J - 3.75)   multiply each side by 3.75

$$N = \frac{3.75(6A)}{(A - 6)(J - 3.75)}$$ replace J with this

$$N = \frac{3.75(6A)}{(A - 6)[[(6A)/(A - 6)] - 3.75]}$$ replace J with this

$$N = \frac{3.75(6A)}{(6A) - (3.75)(A - 6)}$$ multiply thru and cancel

$$N = \frac{22.5A}{6A - 3.75A + 22.5}$$ multiply thru parentheses

N = (22.5A)/(2.25A + 22.5)   combine like terms

J = (6A)/(A - 6)

$$\frac{3}{N} + \frac{3}{J} + \frac{3}{A} = 1$$

$$\frac{3(2.25A + 22.5)}{(22.5A)} + \frac{3(A - 6)}{(6A)} + \frac{3}{A} = 1$$

[make substitutions]

   12(2.25A + 22.5) + 45(A - 6) + 270 = 90A

[multiply thru by 90A and cancel as you go thru]

 27A + 270 + 45A - 270 + 270 = 90A   multiply thru

 72A + 270 = 90A   combine like terms

 -72A   - 72A   subtract 72A from each side
_____

```
    270 = 18A                          subtract
   _____   ____
    18      18          divide each side by 18

   15 = A            divide and cancel

J = (6*15)/(15 - 6)     replace A with 15

J = (90)/(9)             multiply and subtract

J = 10                  divide

  3       3       3
_____  +  ____ +  ____  = 1  make substitutions
  N       10      15

(3/N) + [(9 + 6)/30] = 1     add fractions

(3/N) + (1/2) = 1            add and reduce

3/N = 1/2        subtract 1/2 from each side

N = 6            cross multiply

results:  N = 6;  J = 10 ;  A = 15
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